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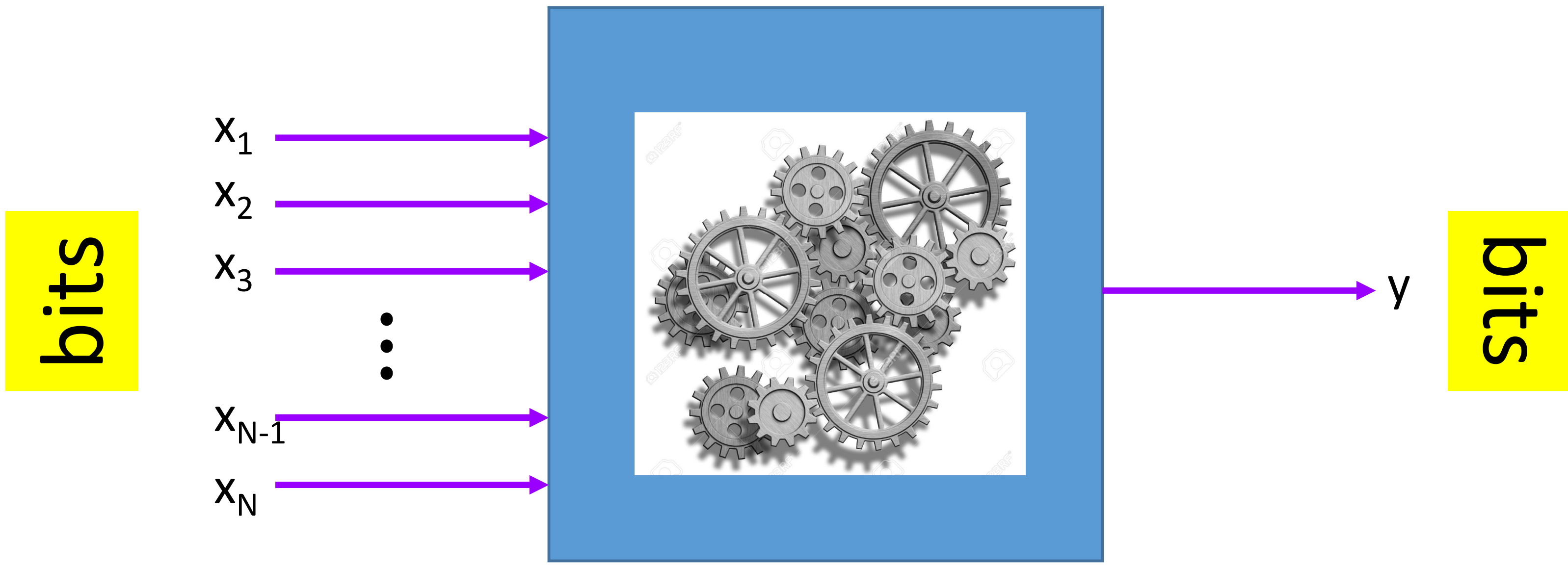
Quantum…

Computing

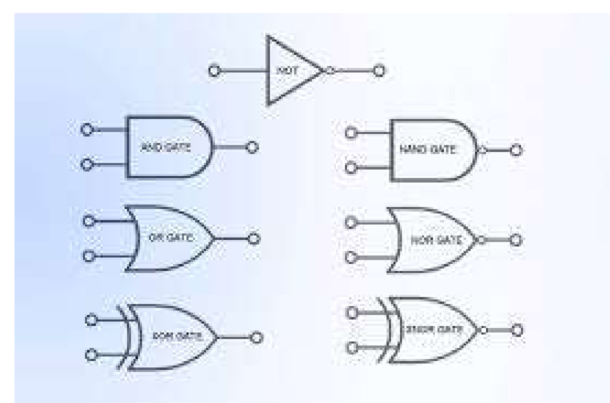
Date: XX XXX XXXX….

Lecture 2: The cat!

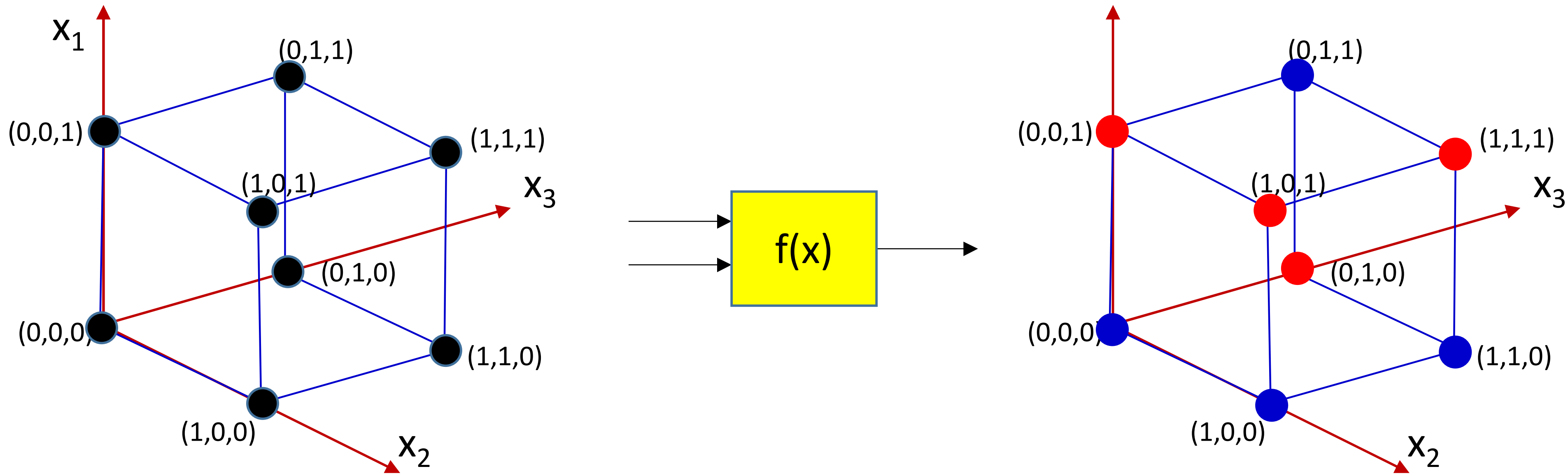
Recap: A model for computation



- A bunch of bits go in, and one bit comes out.
- Examples to the right

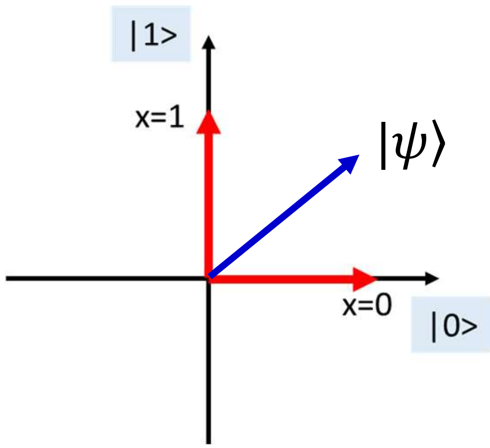


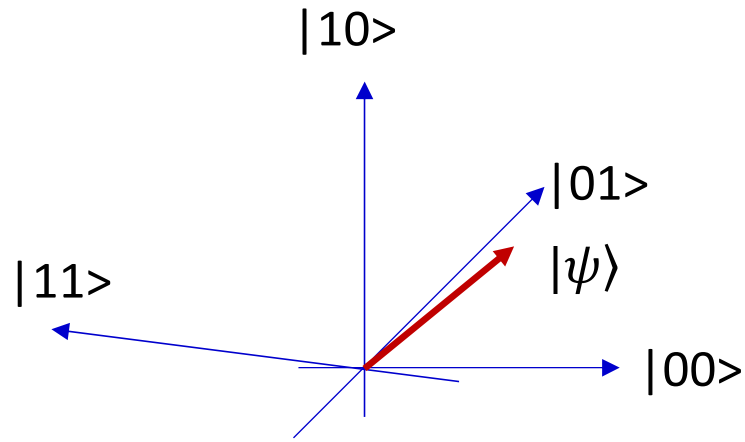
Recap: Classical math



- Algorithms are just functions that operate on bit patterns and produce an output
- Classical approach: For an N-bit input, the function operates on an N-bit input space
    - Each valid bit pattern is a vector in this space
- To fully characterize a unknown glass-box algorithms we must evaluate it on all 2^N feasible inputs
    - Very expensive

Recap: A new binary math

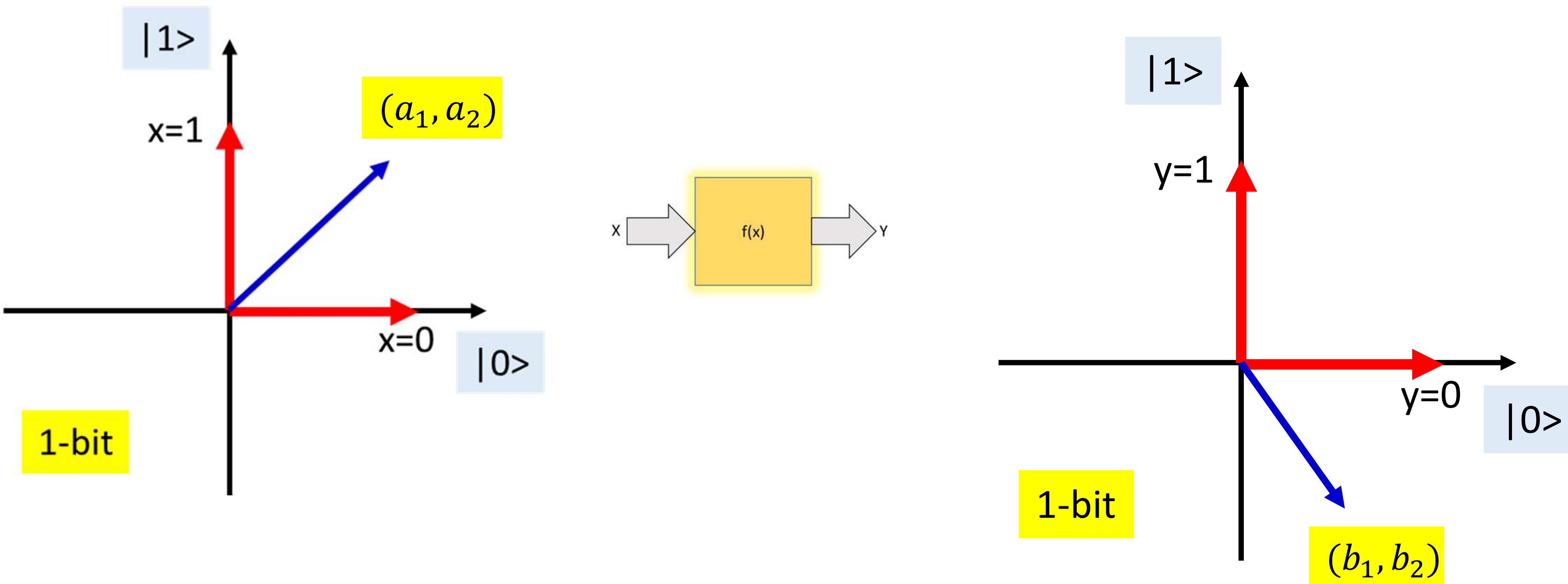




Lame attempt at visualizing 4D

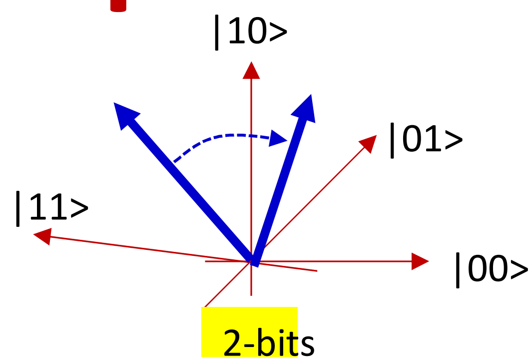
- *Bit patterns* now represent orthogonal directions
- An input is now a vector (a phasor) in this new space
  - And represents a linear combination of bit patterns
  - 1 bit: $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$
  - 2 bits: $|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$
- ***Superposition*** of all possible bit patterns

Recap: The new "quantum" math



- An algorithm is now an operator that operates on the vector to produce another vector
  - Can now compute the output for *all* bit patterns in a single evaluation step
- Caveats – the operator must be:
  - Linear
  - Invertible
  - And not increase the length of the vector (i.e. it must be a rotation)
- **Additional clause**: The "Qbit" phasors must be unit length

Recap: It's not realizable on a classical computer



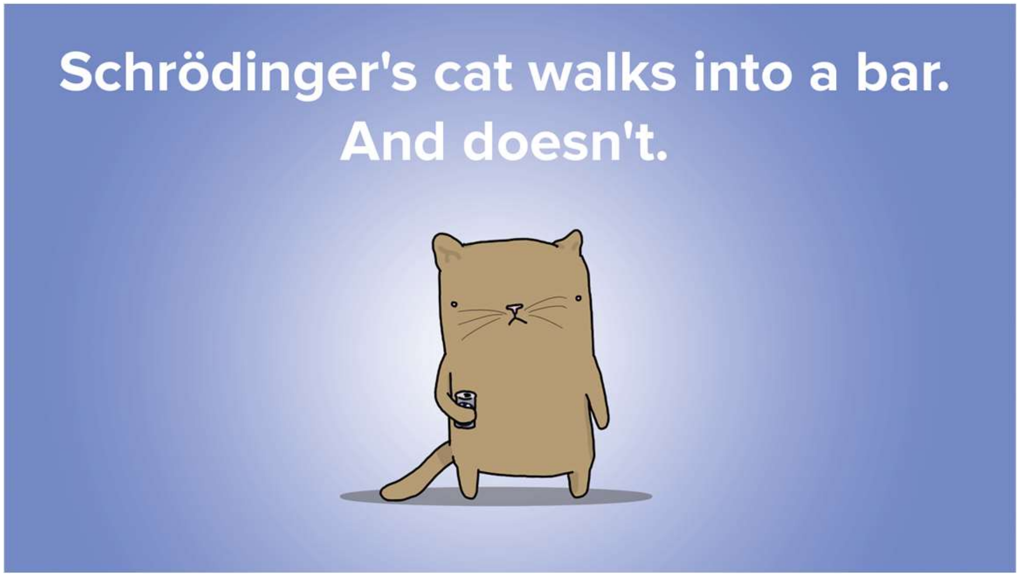
Fact that may only interest me: "Graham's number" is a number that's so large there isn't enough space in the universe to write it..

- A conventional (classical) computer requires a *single* N-bit register to represent a value
  - A function takes in a single N-bit value and produces a single bit

- The new representation requires 2^N numbers to represent a single value!
  - For even N=100 bits, this requires ~1000000000000000000000000000000 numbers
  - A function take in 2^{100} values and produces 2^{100} values
    - I.e it's a 2^{200}-valued transform!
  - Which is why its not a very *useful* way of thinking about things

- Is there a *parsimonious* way of representing a 2100 component vector without taking up the entire universe?

Enter.. The cat!!



- Introducing Quantum, the cat

The world according to Schroedinger

$$ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$$

- The magical formula that represents the quantum number 42

The world according to Schroedinger

$$ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$$

- The magical formula that represents the quantum number 42
- The solution to this is the wave function!
  - For any physical entity you have a wave function and a Hamiltonian
    - The Hamiltonian, which you could design, determines how it evolves over time

- There can be more than one solution to equation
- The entity simultaneously populates *every* solution to the above equation
  - It is in a superposition of all solutions
  - This is particularly true of the smallest units of the universe – the quanta
    - But also of everything else, really

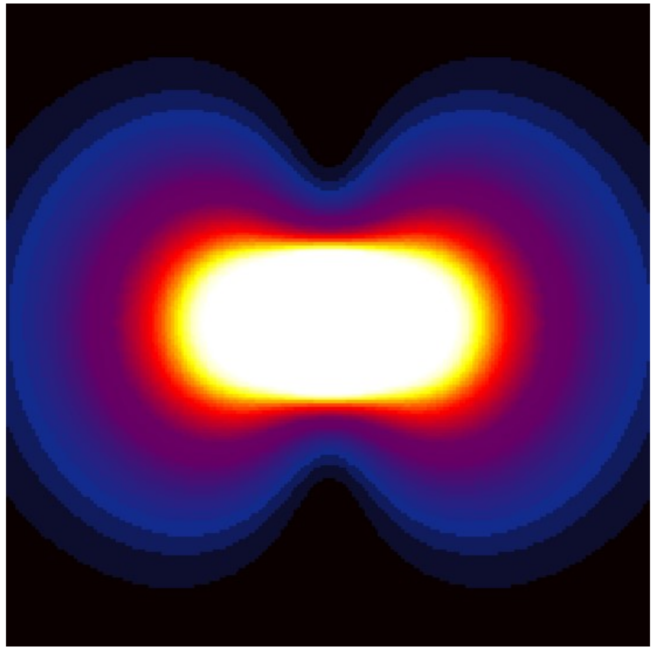
The wave function for any particle predicts its probability

$$ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle \Rightarrow \psi(t,x)$$

$|\psi(t,x)|^2$ is the probability that the system will be *found to be* in configuration x at time t, if we were to 'measure' it!!

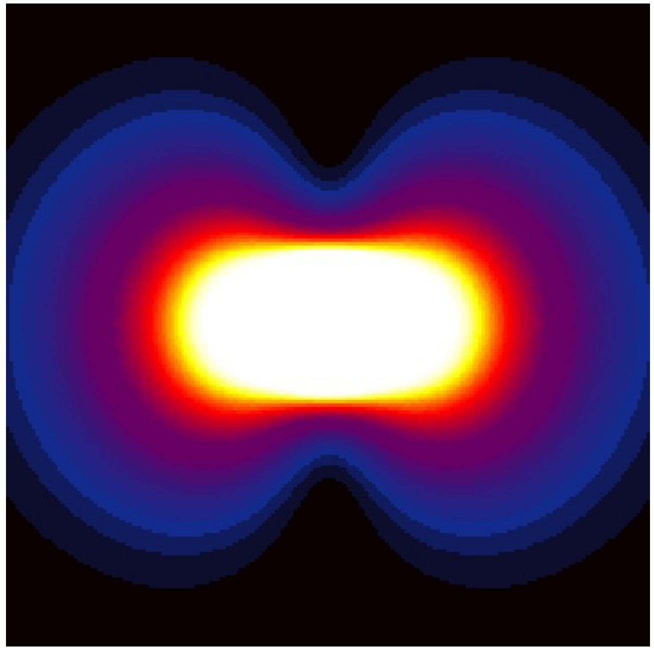
- What does this mean
- Many (possibly an infinity of) solutions exist to the equation
  - One for each x
- The entity lives in a *superposition* of all options: $\sum_x \psi(t,x)|x\rangle$ or $\int_x \psi(t,x)|x\rangle dx$
  - $\psi(t,x)$ is a **complex** value associated with each solution x at time t
- But if you were to try to *observe* it, only one of these solutions would be observed
  - The probability of observing solution x at time t is given by $|\psi(t,x)|^2$

The hydrogen atom



- A hydrogen molecule consists of two atoms sharing two electrons
- **Consider one of these electrons**
  - **Where is it at any time? Around which nucleus in particular?**

The hydrogen atom



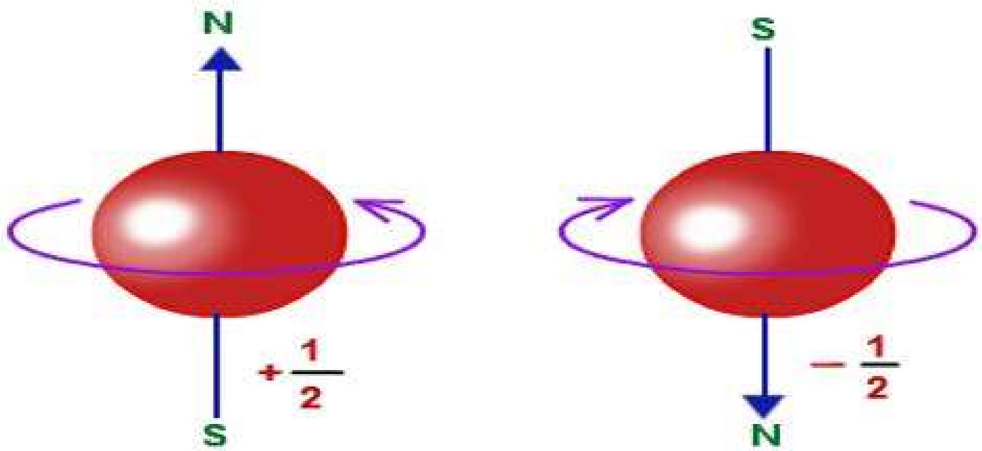
$$\psi(t, x)$$

$$\psi(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle$$

x = |0> and x = |1>
are the two solutions

- When you are not looking for it, it is *actually* simultaneously at both nuclei
  - **It is in a *superposition* of both states**
- $|\alpha|^2$ and $|\beta|^2$ are the probability of *finding* the electron at each location in the hydrogen molecule

The spin of an electron

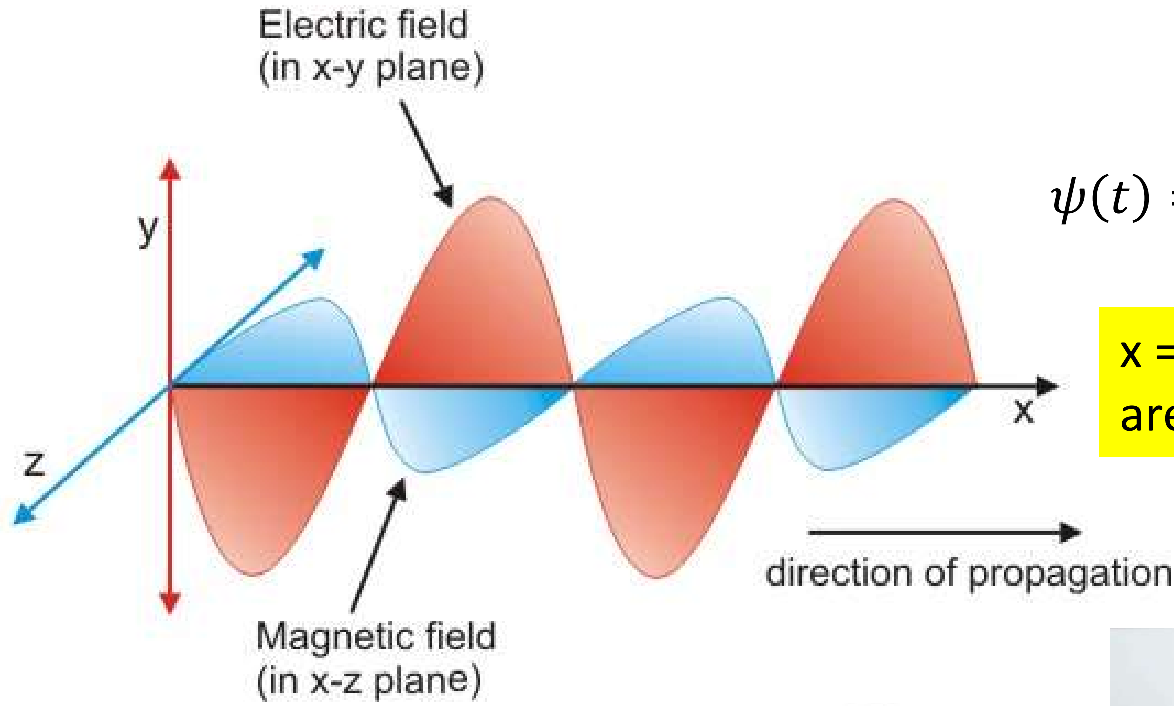


$$\psi(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle$$

x = |0> and x = |1>
are the two solutions

- When you are not looking for it, it is *actually* simultaneously spinning both ways
  - **It is in a *superposition* of both states**
- $|\alpha|^2$ and $|\beta|^2$ are the probability of *finding* the electron in each spin state

The polarization of light

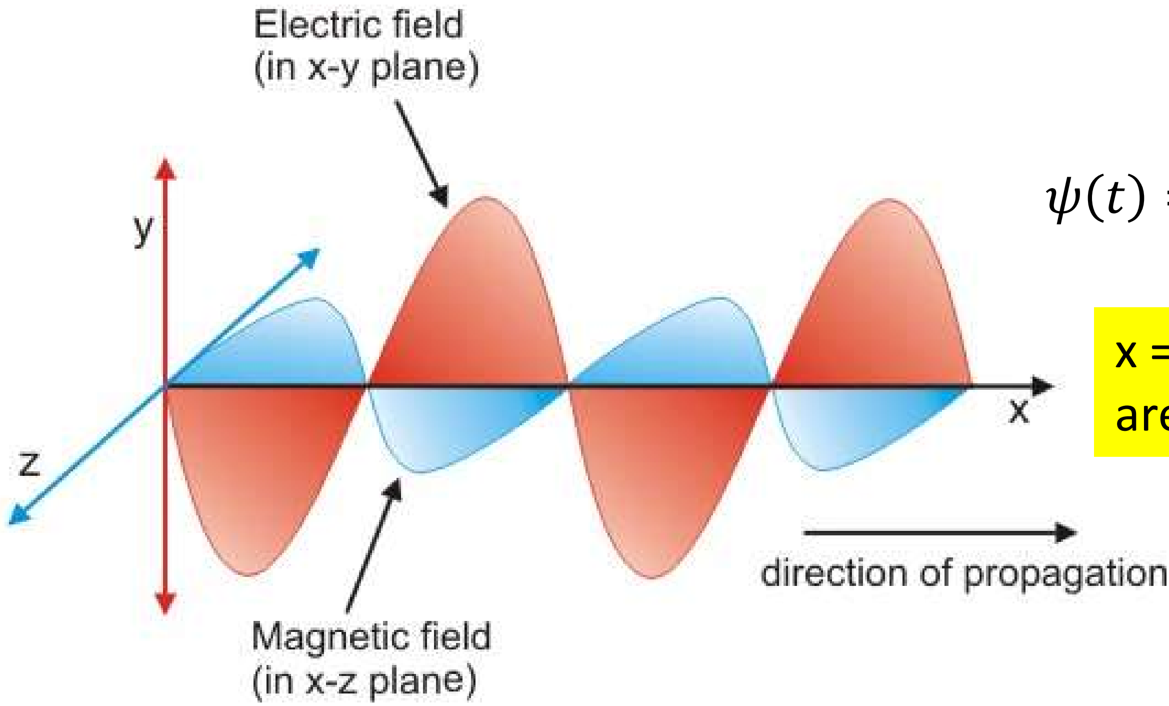


$$\psi(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle$$

x = |0> and x = |1>
are the two solutions

- How do polarizing glasses work?
  - Send through the component of the E/M fields that are aligned with the polarizer

The polarization of light

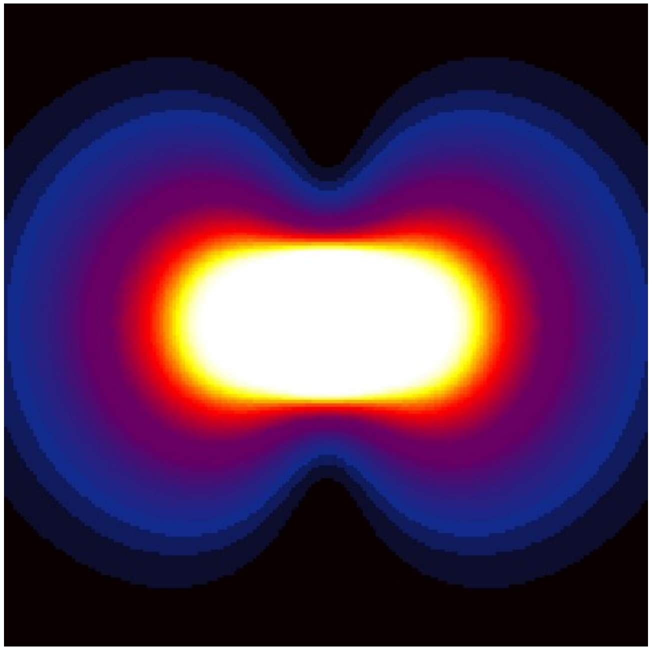


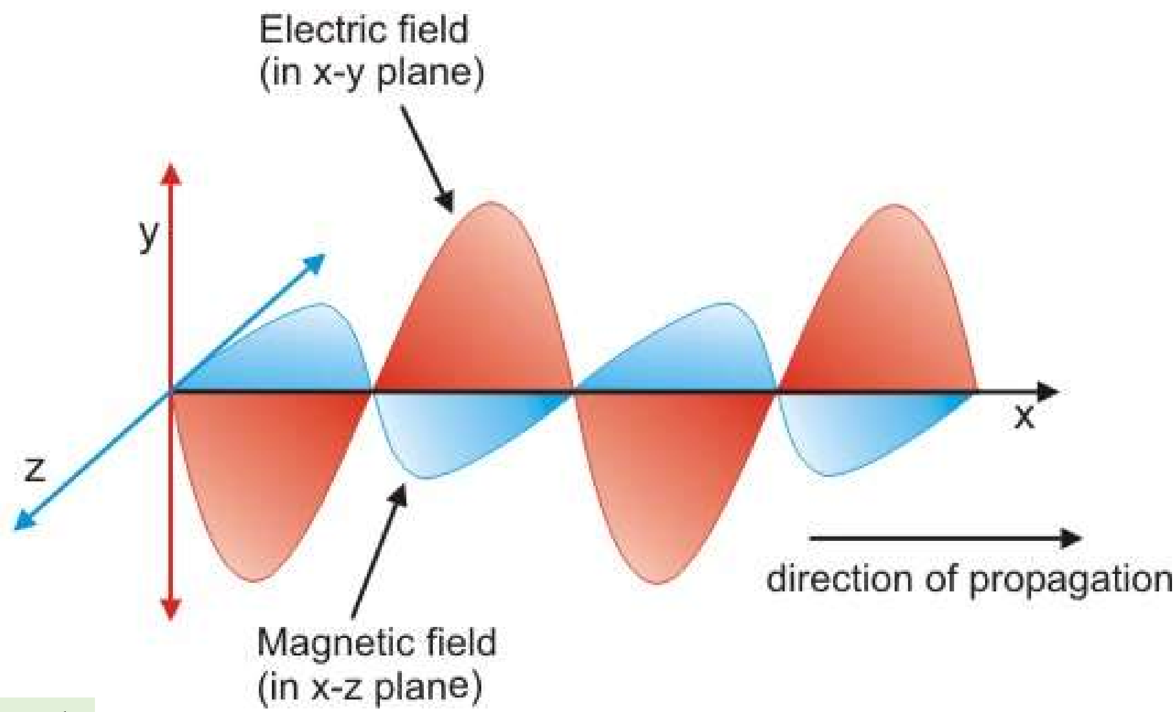
$$\psi(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle$$

x = |0> and x = |1>
are the two solutions

- The light actually exists in both polarizations at the same time
  - ***It is in a superposition of both polarizations***
- The polarizing glass ‘observes’ (or *measures*) the photons, and they end up as one or the other

Quantum systems





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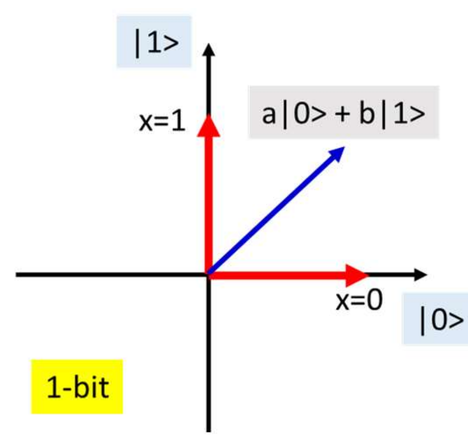
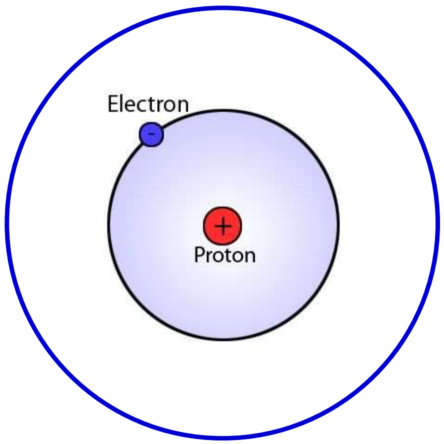
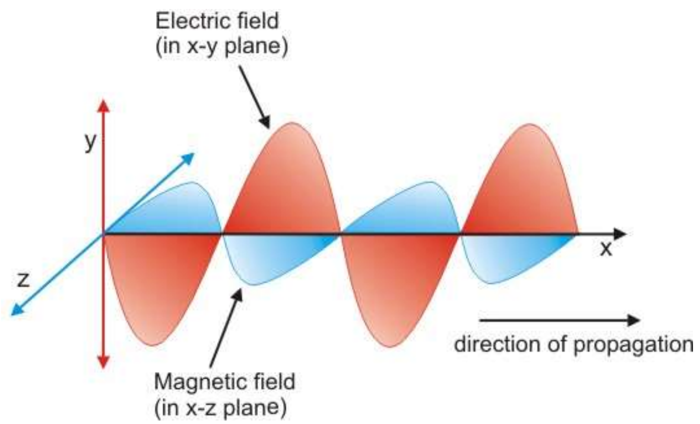
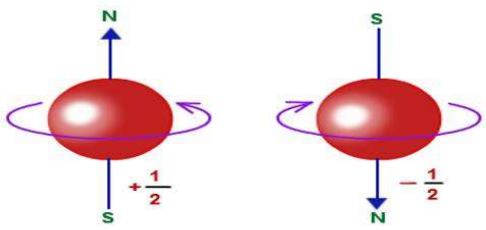


- Quantum systems naturally exist in a superposition of multiple values
  - If we assign each value to a bit value (or bit pattern), we get a quantum computing platform

Recap: Implementing the qubit







- Cannot use classical physics
  - Will require computers with exponential amounts of memory to represent even a small number of bits

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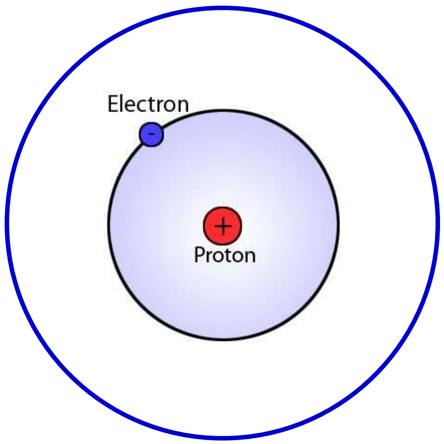
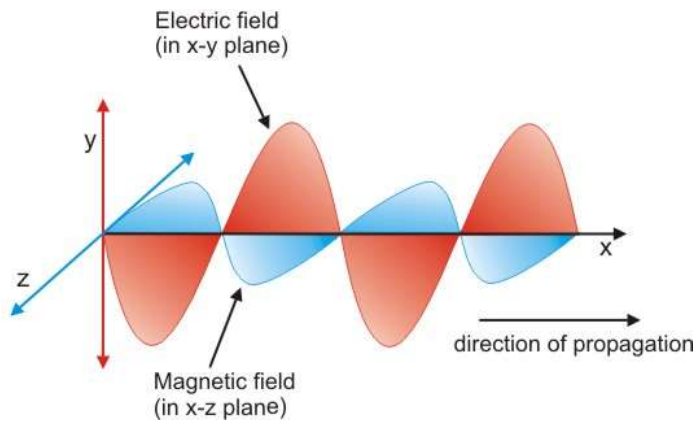
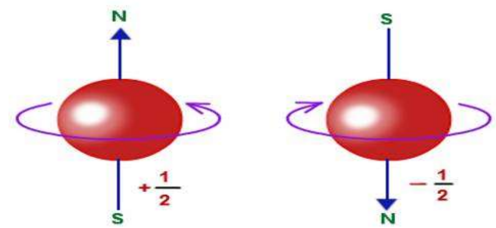


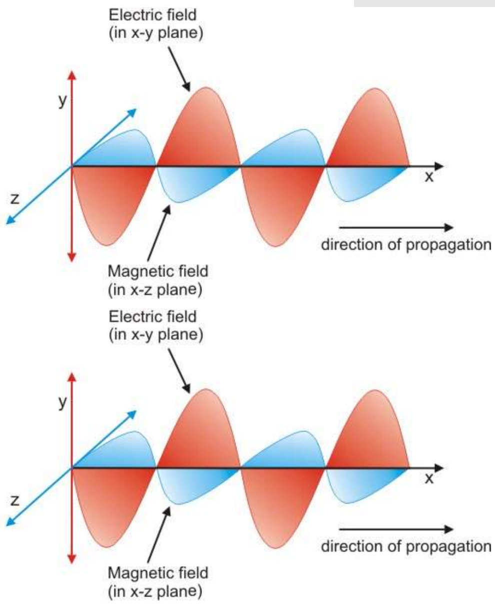


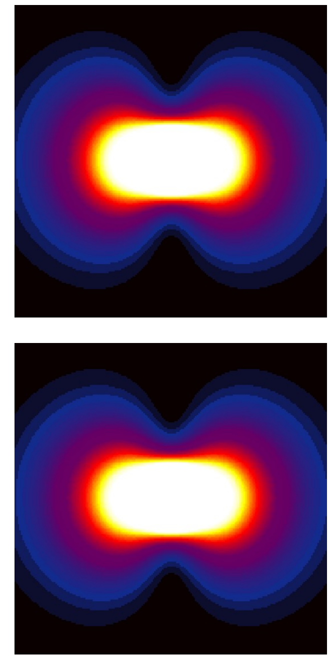


- Cannot use classical physics
  - Will require computers with exponential amounts of memory to represent even a small number of bits
- Use quantum physics
  - Derived from Schrodinger's equation: $ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$
    - Every particle is a wave that exists in all states $|\psi(t,x)\rangle$ simultaneously
    - $|\psi(t,x)\rangle|^2$ = probability of finding the system in configuration x at time t when you measure it
  - Use quantum properties of quantum particles to implement the bit
  - E.g: The energy level of an electron
  - E.g: The spin of an electron
  - E.g: The polarization of a photon

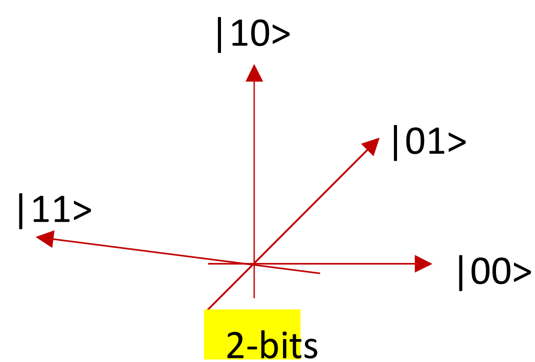
Multiple bits

a|00> + b|01> + c|10> + d|11>

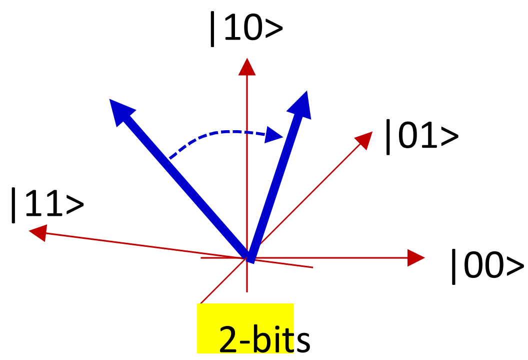




- Increasing the number of bits only takes increasing the number of basic quantum units



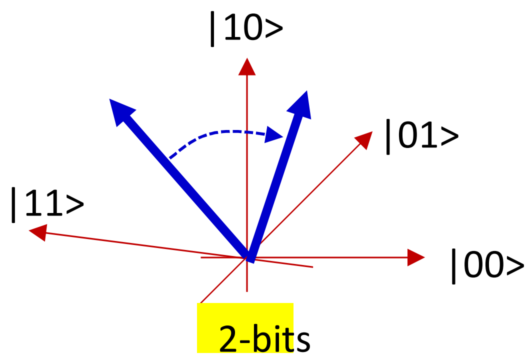
Practical implementation



- Simply use a collection of quantum bits
  - Will simultaneously represent all states

- What is missing?

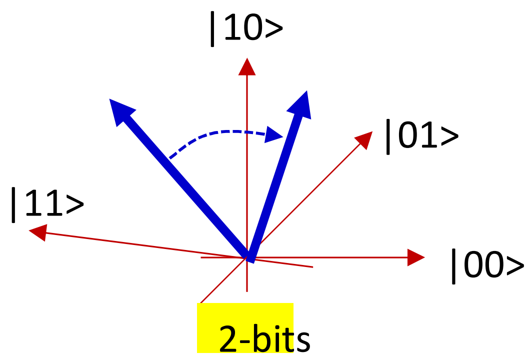
Practical implementation



- Simply use a collection of quantum bits
  - Will simultaneously represent all states
- What is missing?
  - How do you implement the functions?
    - Invertible rotations $ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$

But first you must design the functions (we will see how)

Practical implementation



- Simply use a collection of quantum bits
  - Will simultaneously represent all states
- What is missing?
  - How do you implement the functions?
    - Invertible rotations $ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$
  - How do you measure the output vectors?

The wave function for any particle predicts its probability

$$\psi(t, x)$$

$|\psi(t, x)|^2$ is the probability that the system will be *found to be* in configuration x at time t, if we were to 'measure' it!!

- What does this mean
- Many (possibly an infinity of) solutions exist to the equation
  - One for each x
  - $\psi(t, x)$ is a complex value associated with each solution x at time t
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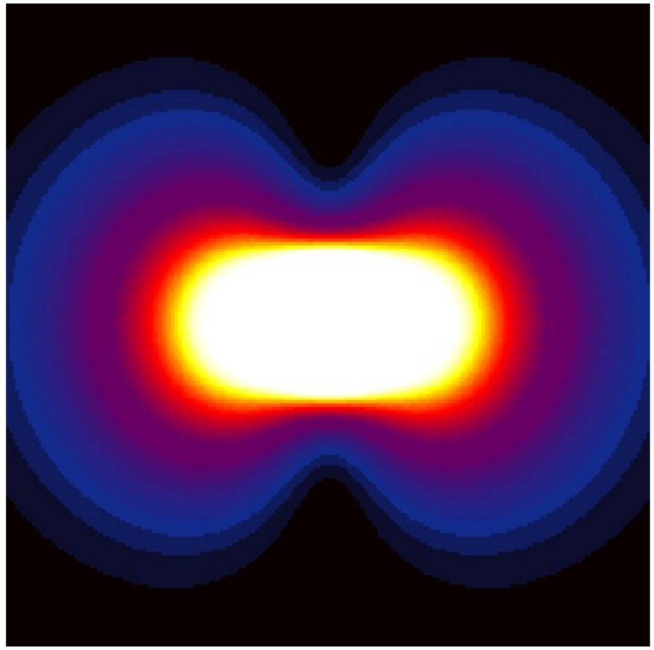
The hydrogen atom

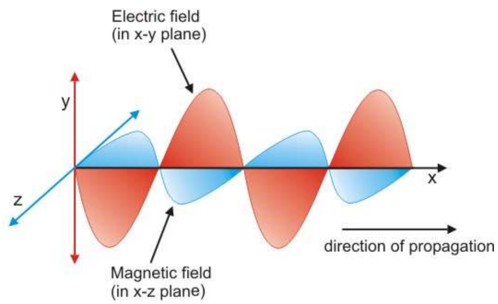


$$\psi(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle$$

- $|\alpha|^2$ and $|\beta|^2$ are the probability of ***finding*** the electron at each location in the hydrogen molecule
  - When you are not looking for it, it is *actually* simultaneously at both atoms

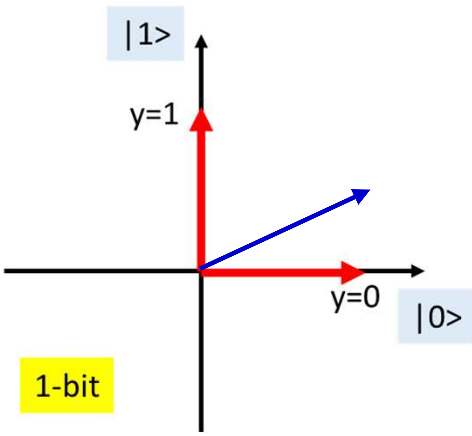
The problem with measurement

- **Reality Doesn't Exist Until We Measure It, Quantum Experiment Confirms**
- https://www.sciencealert.com/reality-doesn-t-exist-until-we-measure-it-quantum-experiment-confirms

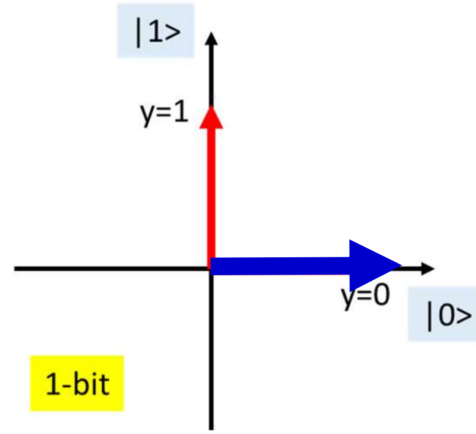


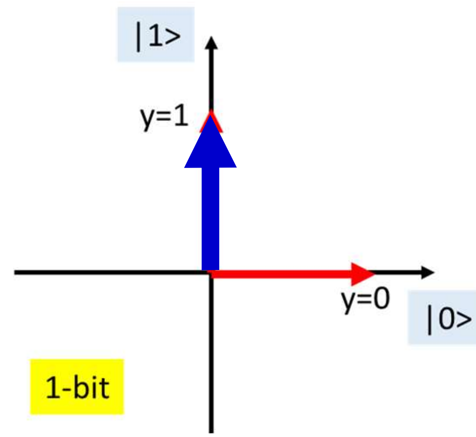
- Measuring a quantum variable "collapses" it

Measurement



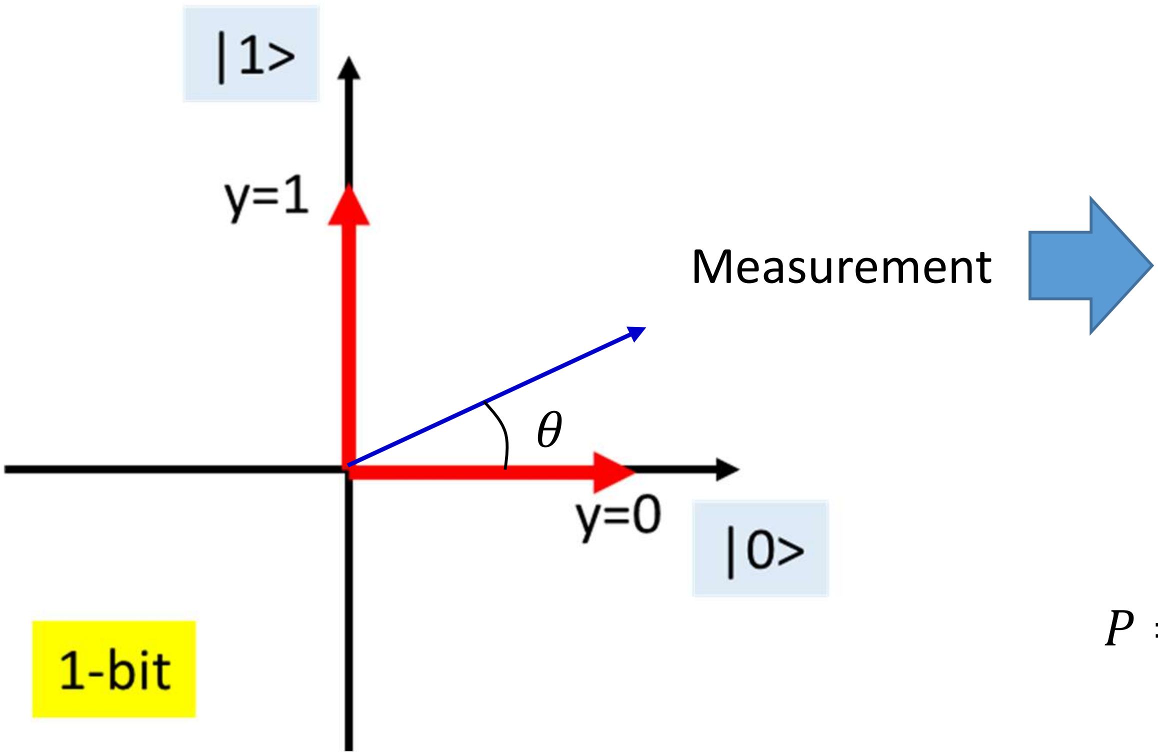
Measurement

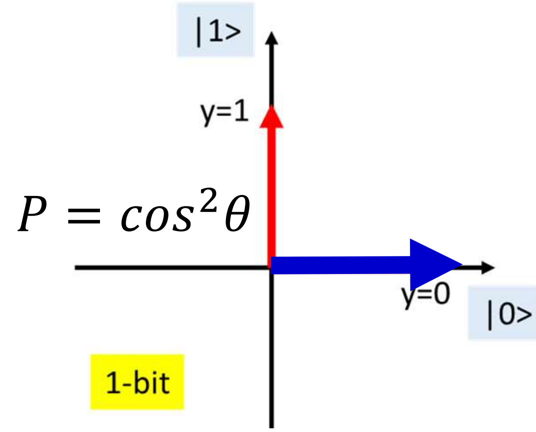


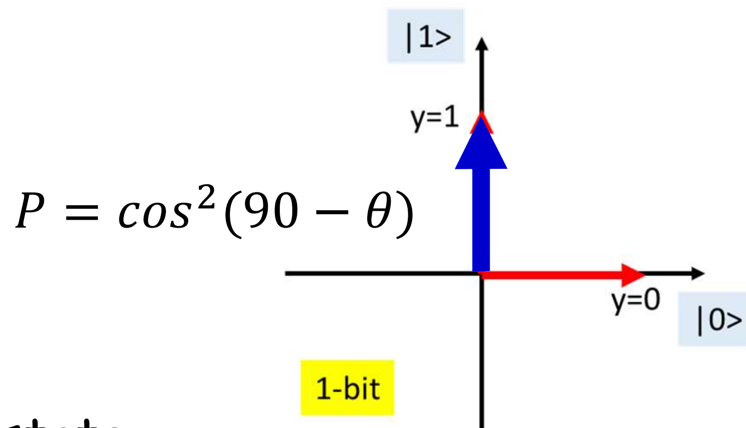


- Measuring the output *collapses* the vector to one of the states
  - Bit pattern
- Which one

Measurement



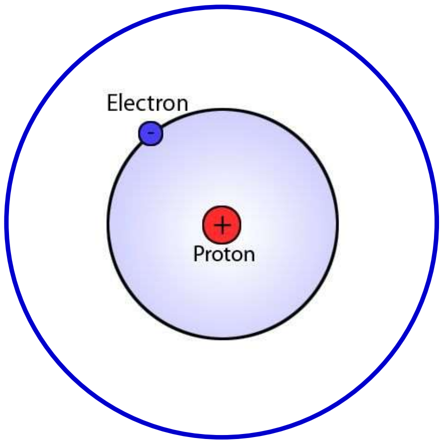


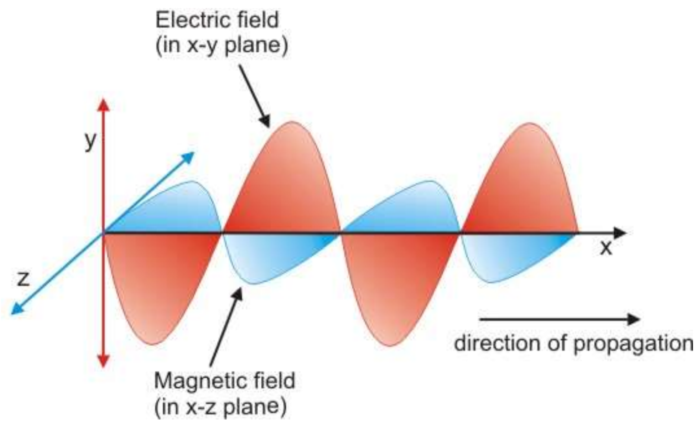


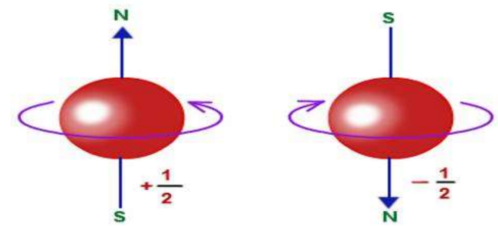
You can never *observe* the superposed state
Any attempt at observation will show up as one state or the other!
"collapsing"

The probability of 'collapsing' to a state depends on the angle of the 'phasor' (the superposed state)

The physical Qubit

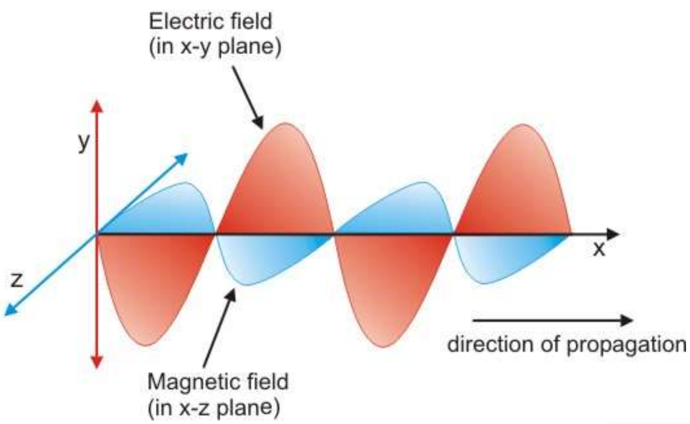


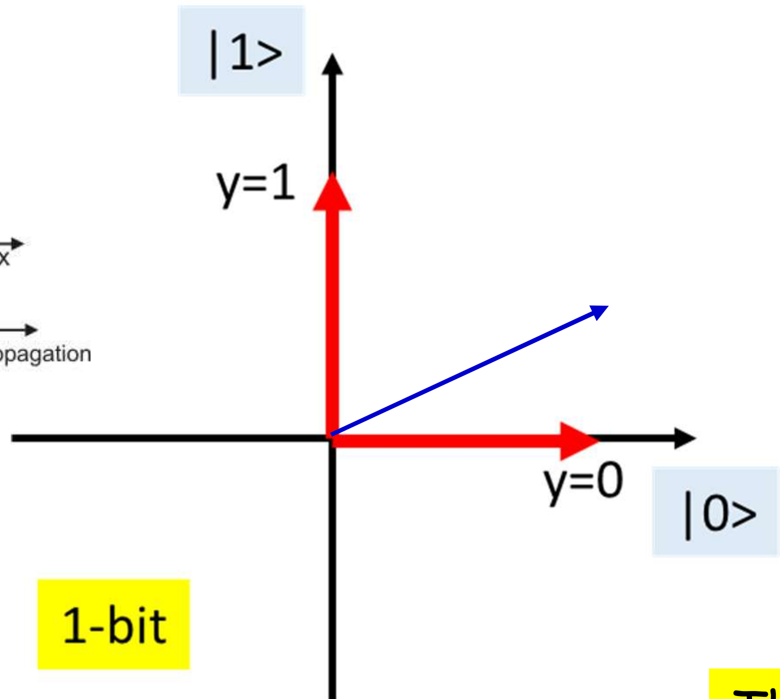




- Hydrogen atom electron:
  - Ground state = |0>, excited state = |1>
- Photon polarization
  - Horizontal = |0>, vertical = |1>
- Electron spin
  - NS = |0>, SN = |1>

The physical Qubit





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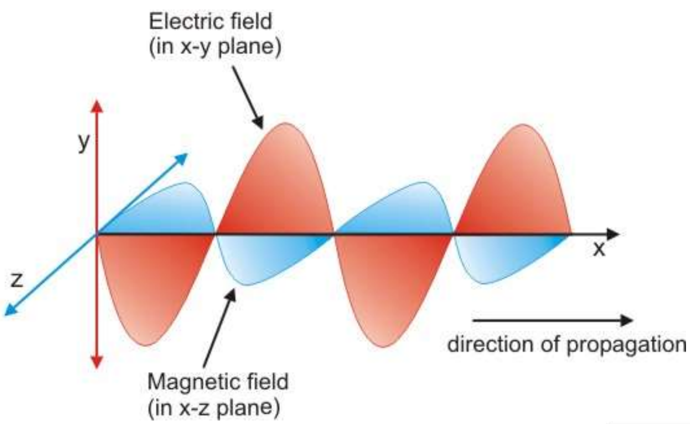
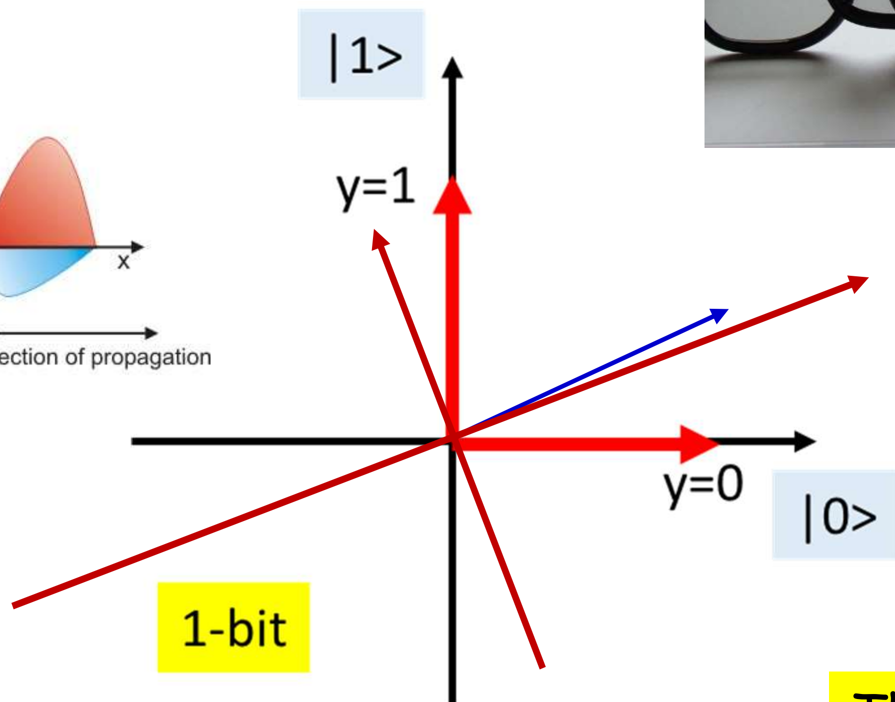
The 'states' are *orthogonal* directions in this space of the wavefunction

Also called 'bases'

The physical Qubit





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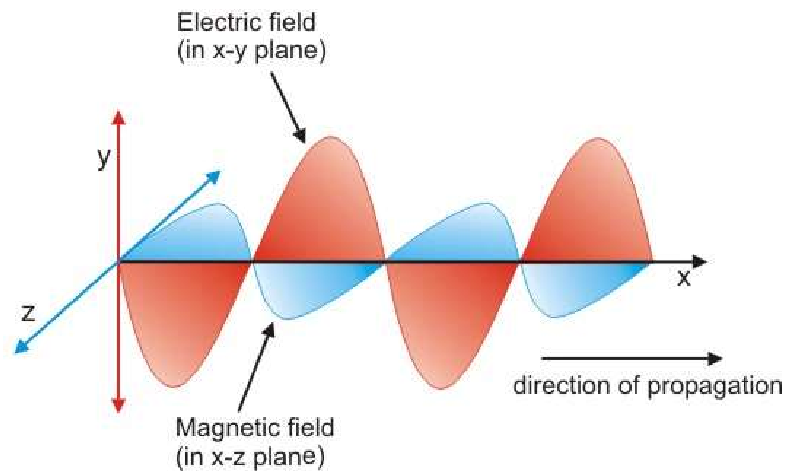
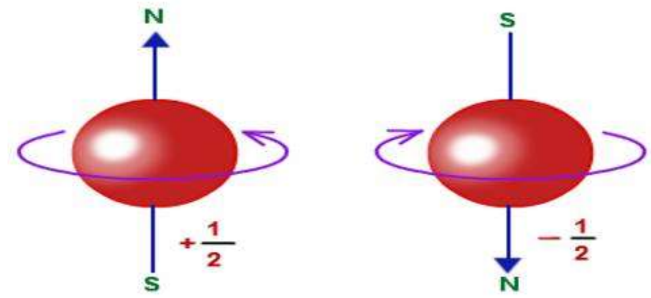
Note: The definition of your "bases" is a matter of convention

The physical Qubit





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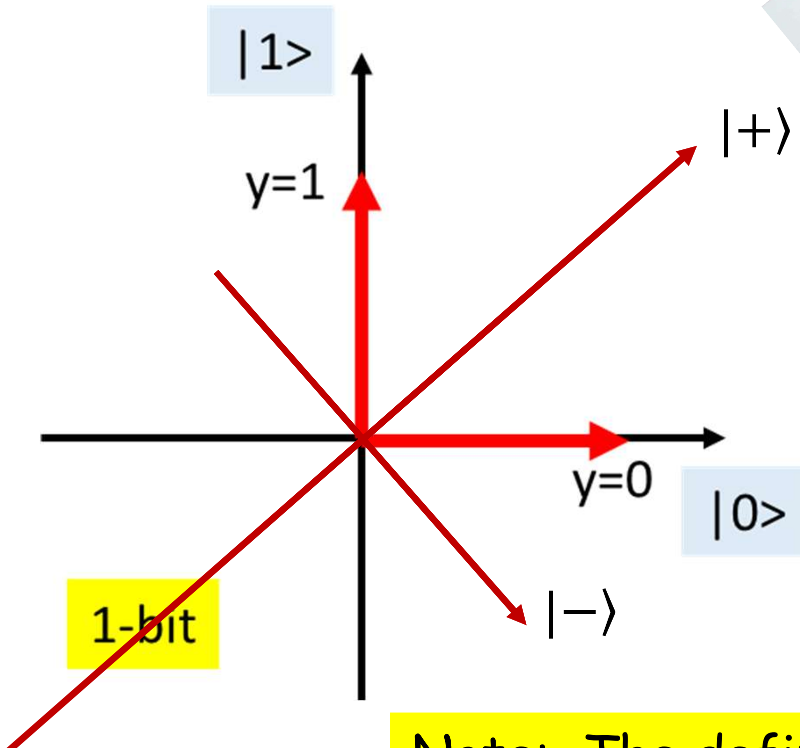

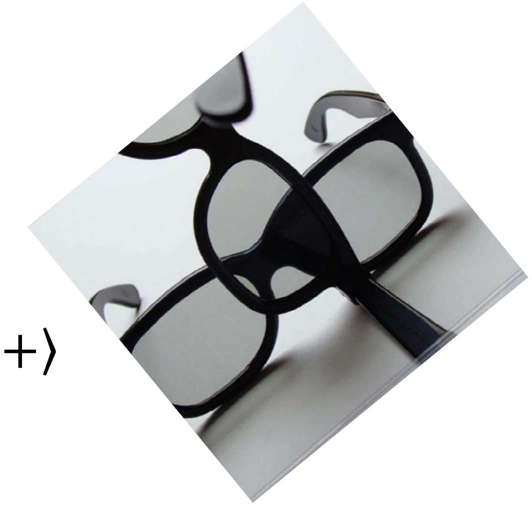

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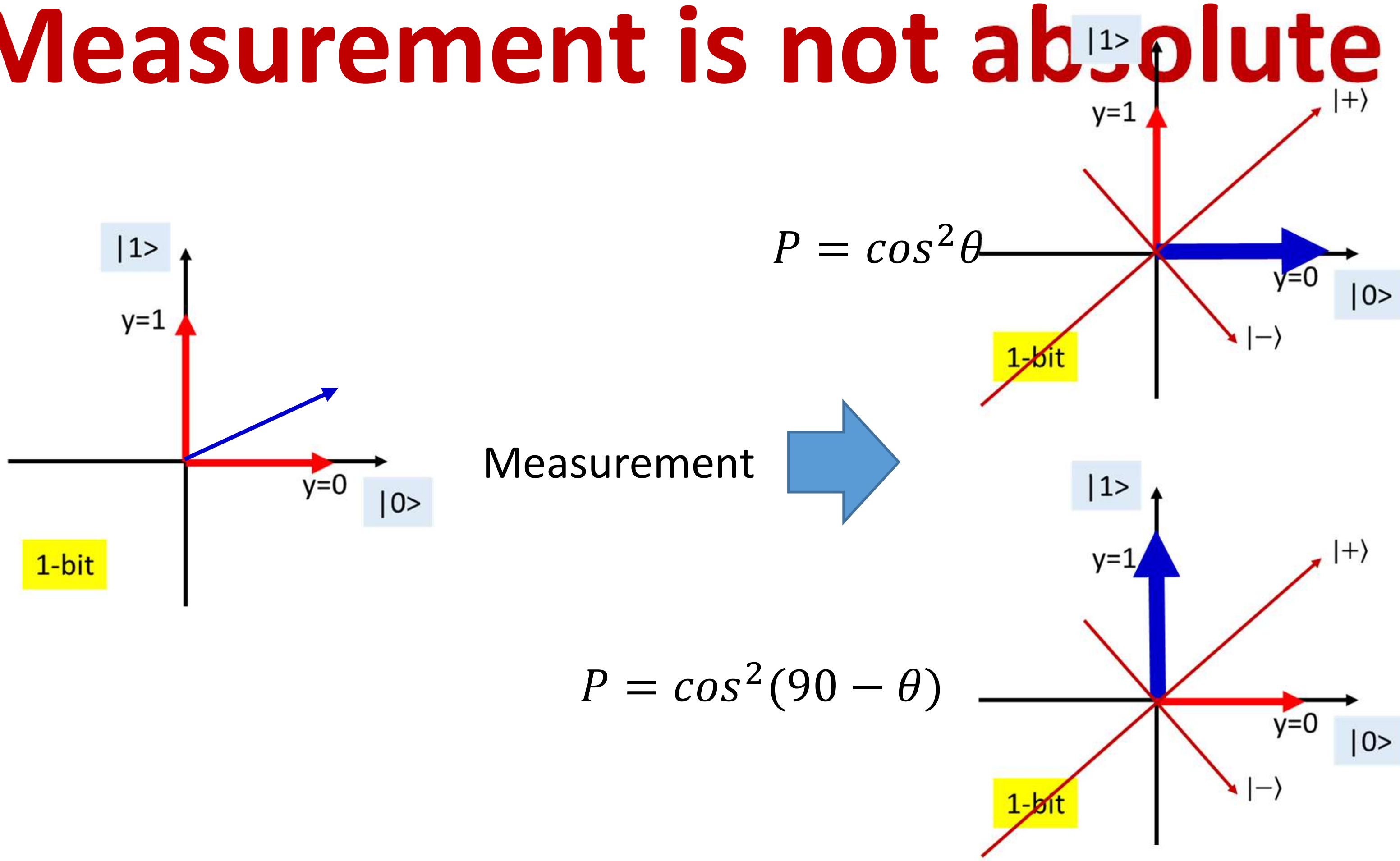
Note: The definition of your "bases" is a matter of convention

You can have *multiple* bases
Designate one as "canonical" and the rest are oriented w.r.t it

The plus/minus bases are commonly invoked

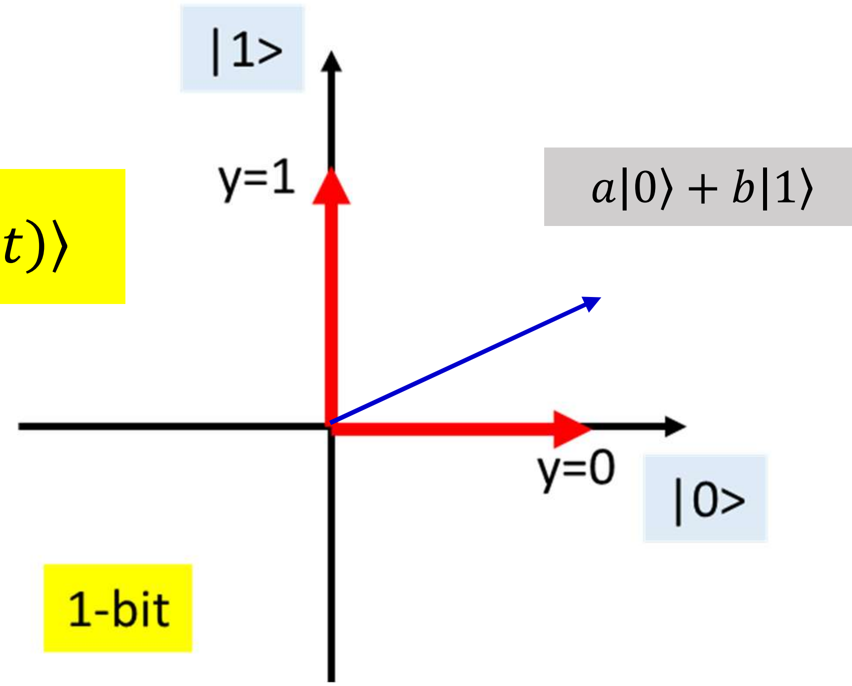
Measurement is not absolute



- Collapsing the vector according to one basis can still keep it indeterminate for other bases!
  - We will use this factor

It’s complex, but not complicated

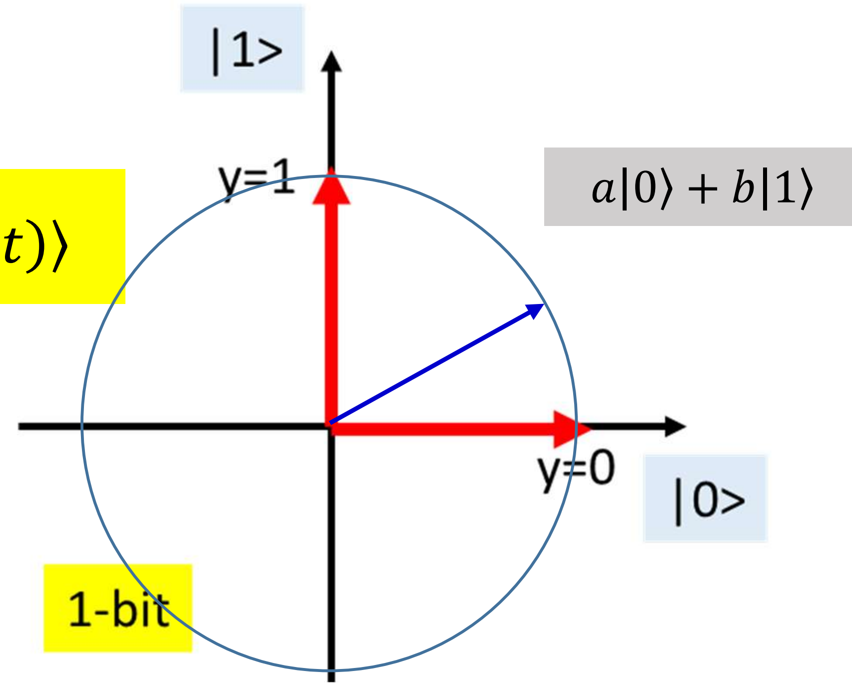
$$ih\frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$$



- The “weights” a and b are actually complex variables
  - Because Schroedinger’s equation describes them as complex
- This simple visualization is *wrong*
  - Its missing two dimensions
    - The imaginary components of a and b

Restrictions on the weights

$$i h \frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$$



- $|a|^2 + |b|^2 = 1$
  - The qubits live on the surface of a hypersphere
- $P(|0\rangle) = |a|^2, \;\; P(|1\rangle) = |b|^2$
- $a = r_a e^{j\gamma}, b = r_b e^{j\delta}$
  - What is the relation between r_a and r_b

Restrictions on the weights

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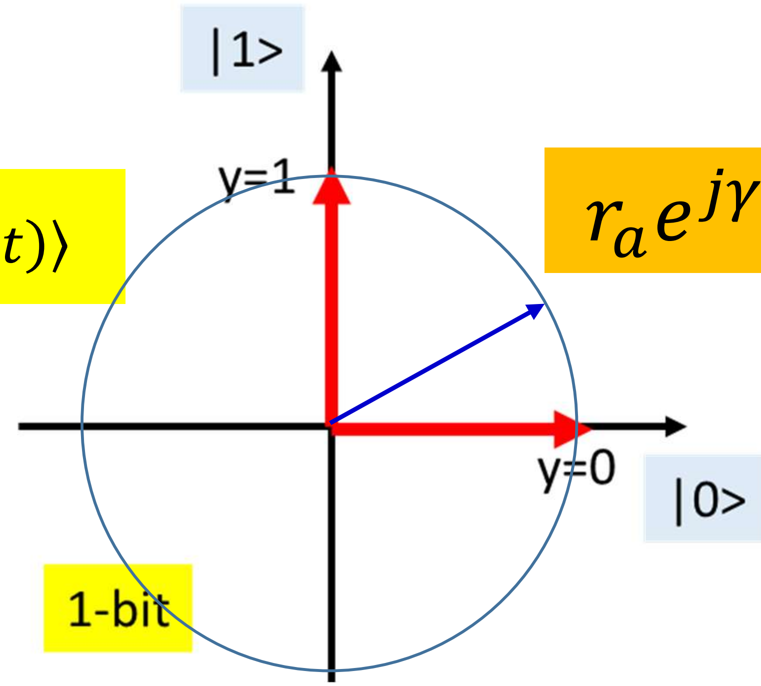

$$r_a e^{j\gamma}|0\rangle + r_b e^{j\delta}|1\rangle$$

- $|a|^2 + |b|^2 = 1$
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Restrictions on the weights

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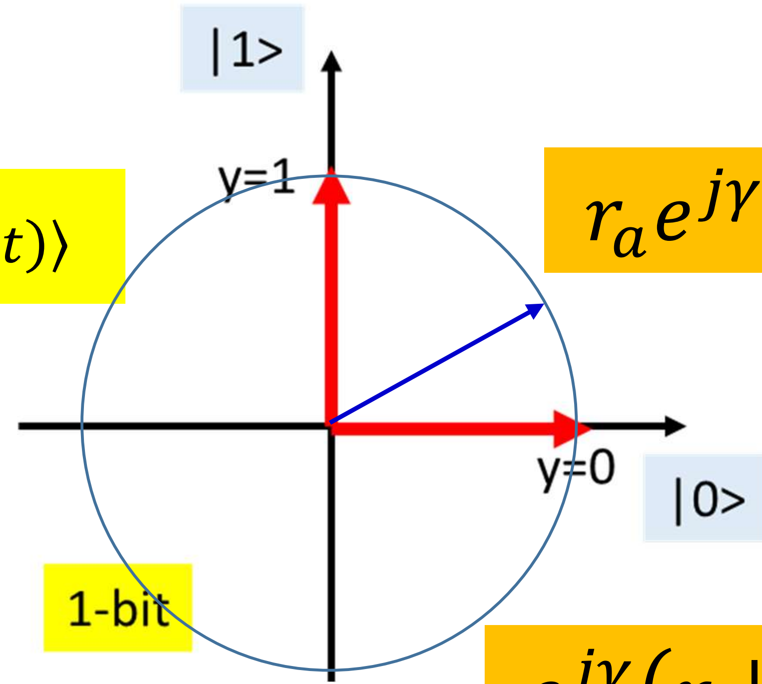


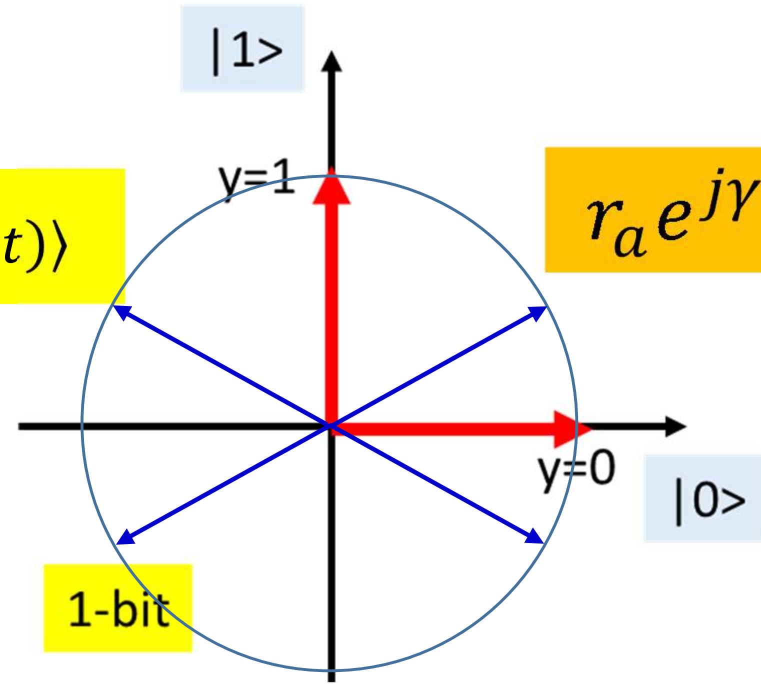
$$r_a e^{j\gamma}|0\rangle + r_b e^{j\delta}|1\rangle$$

$$e^{j\gamma}\left(r_a|0\rangle + r_b e^{j\delta-\gamma}|1\rangle\right)$$

- $|a|^2 + |b|^2 = 1$
  - The qubits live on the surface of a hypersphere
- $P(|0\rangle) = |a|^2, \; P(|1\rangle) = |b|^2$
- $a = r_a e^{j\gamma}, b = r_b e^{j\delta}$
  - What is the relation between r_a and r_b

Something odd

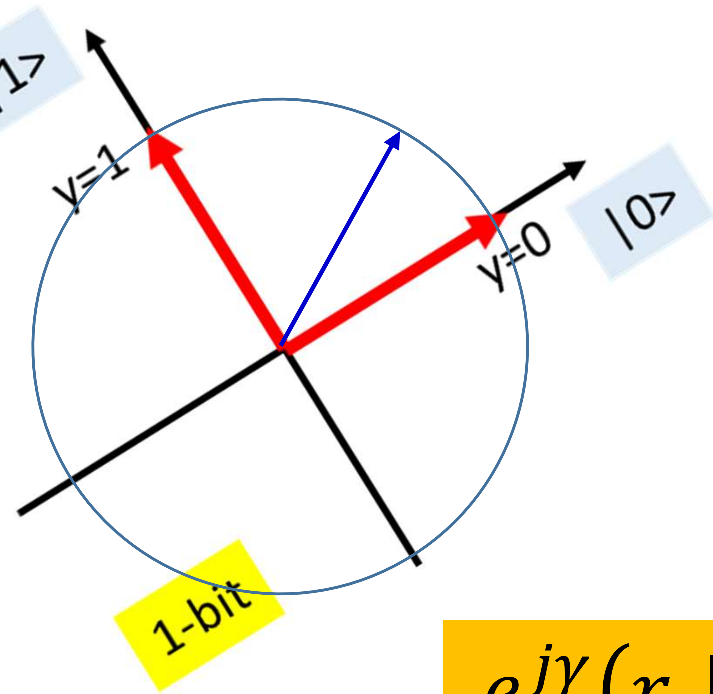
$$i\hbar \frac{d}{dt}|\psi(t)\rangle = 2\pi H|\psi(t)\rangle$$



$$r_a e^{j\gamma}|0\rangle + r_b e^{j\delta}|1\rangle$$

- All of these vectors represent the *same* $P(|0\rangle)$ and $P(|1\rangle)$
- But they're actually different phasors
  - Something we will use all the time

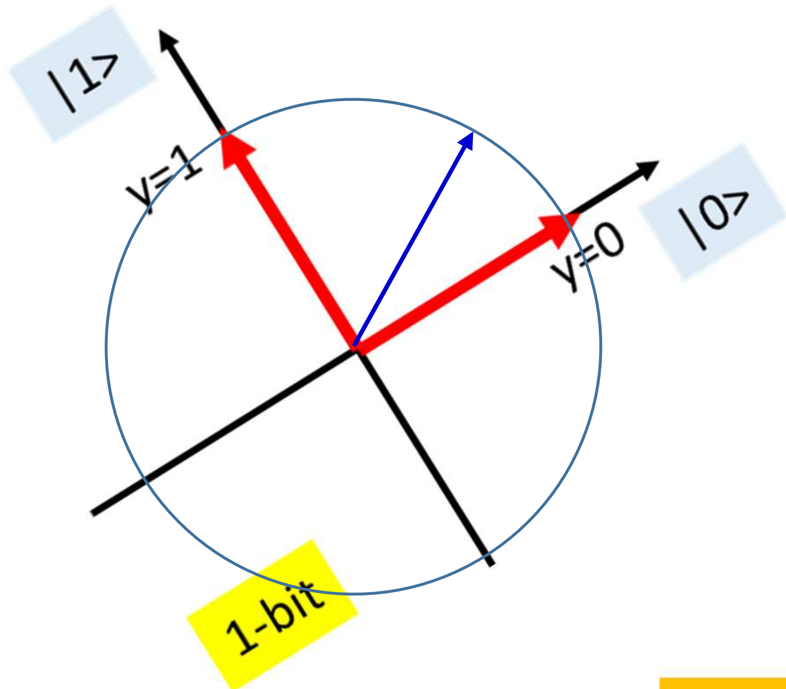
The qubit is rotation invariant



$$e^{j\gamma}\left(r_a|0\rangle + r_b e^{j(\delta-\gamma)}|1\rangle\right)$$

- Rotating the space doesn’t matter

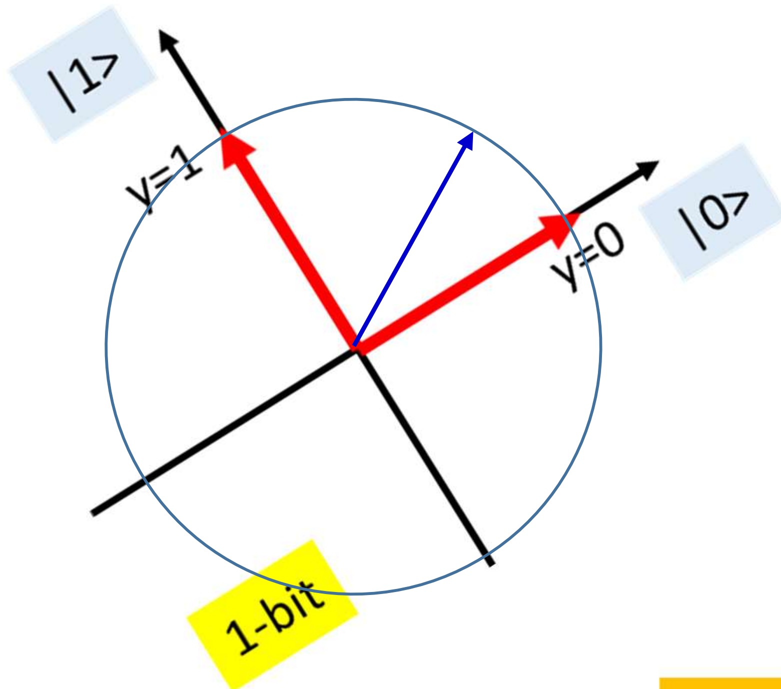
The qubit is rotation invariant



$$r_a|0\rangle + r_b e^{j(\delta-\gamma)}|1\rangle$$

- Rotating the space doesn't matter
  - What is the relation between r_a and r_b

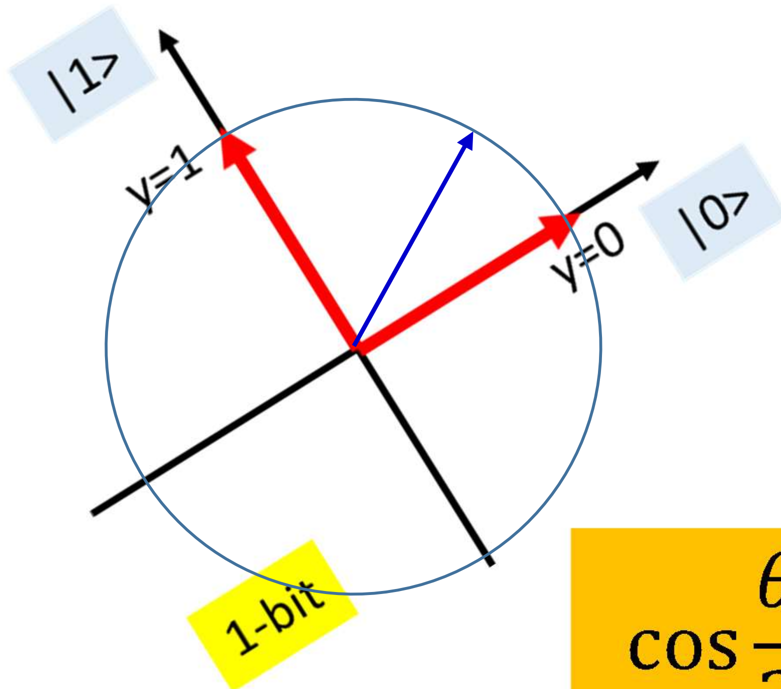
The qubit is rotation invariant



$$r_a|0\rangle + r_b e^{j\phi}|1\rangle$$

- Rotating the space doesn't matter
  - What is the relation between r_a and r_b

The qubit is rotation invariant

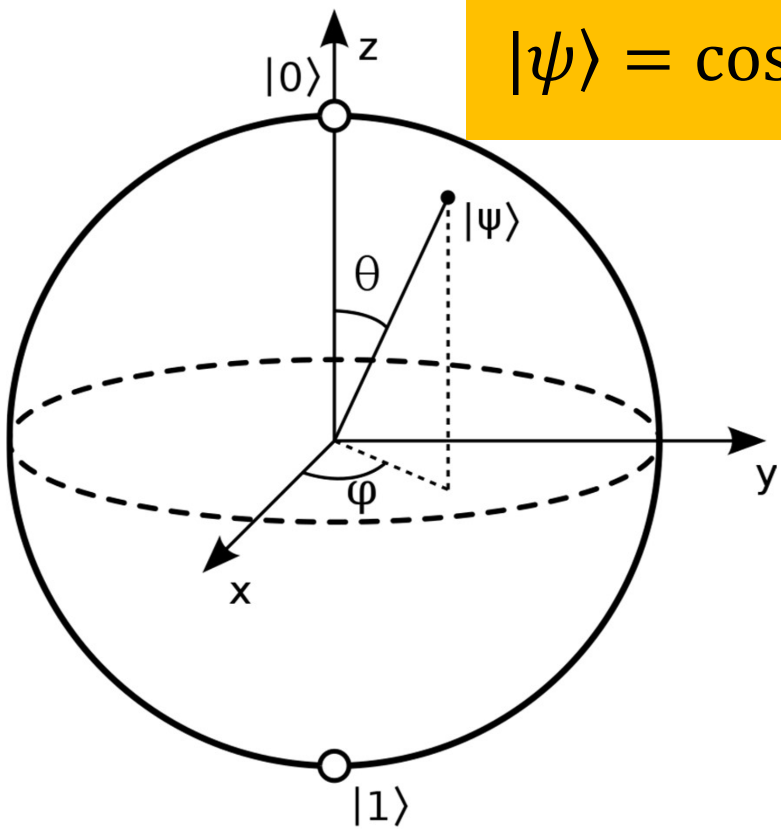


$$\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{j\phi}|1\rangle$$

- Rotating the space doesn't matter
  - What is the relation between r_a and r_b
- This is now a two-variable representation of two variables!
  - Can be visualized

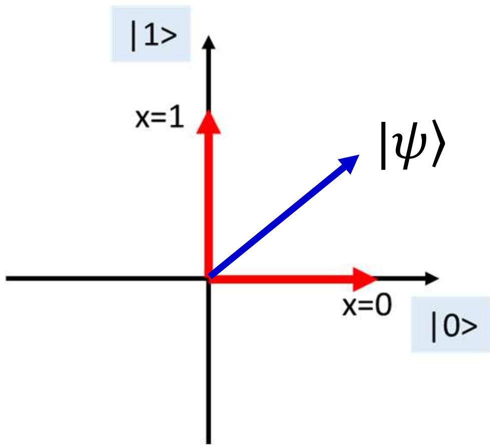
The Bloch Sphere

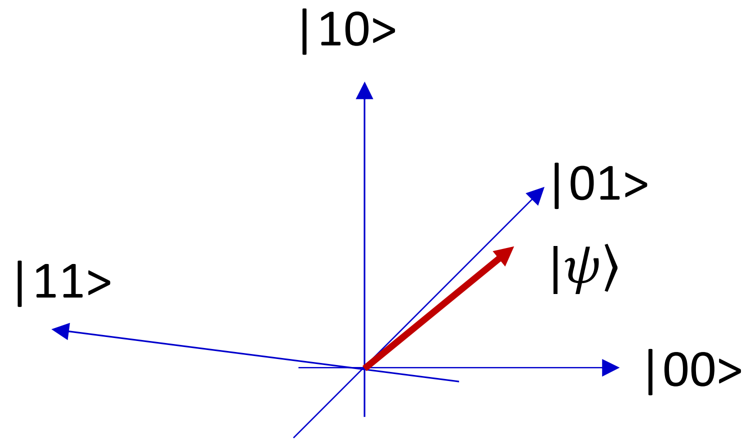
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{j\phi}|1\rangle$$



- Visualizing the qubit
  - 2 variable visualization in a 3D space

Recap: The physical quantum Qubit

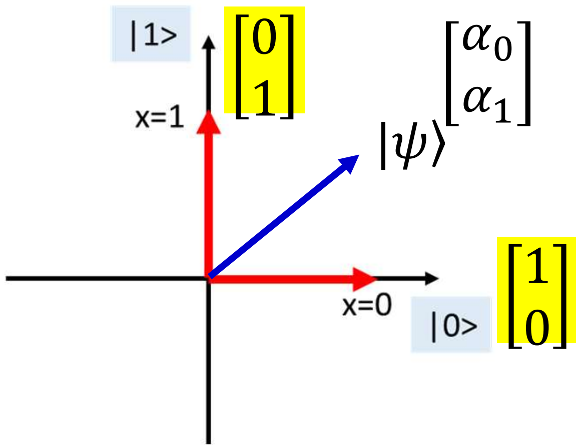


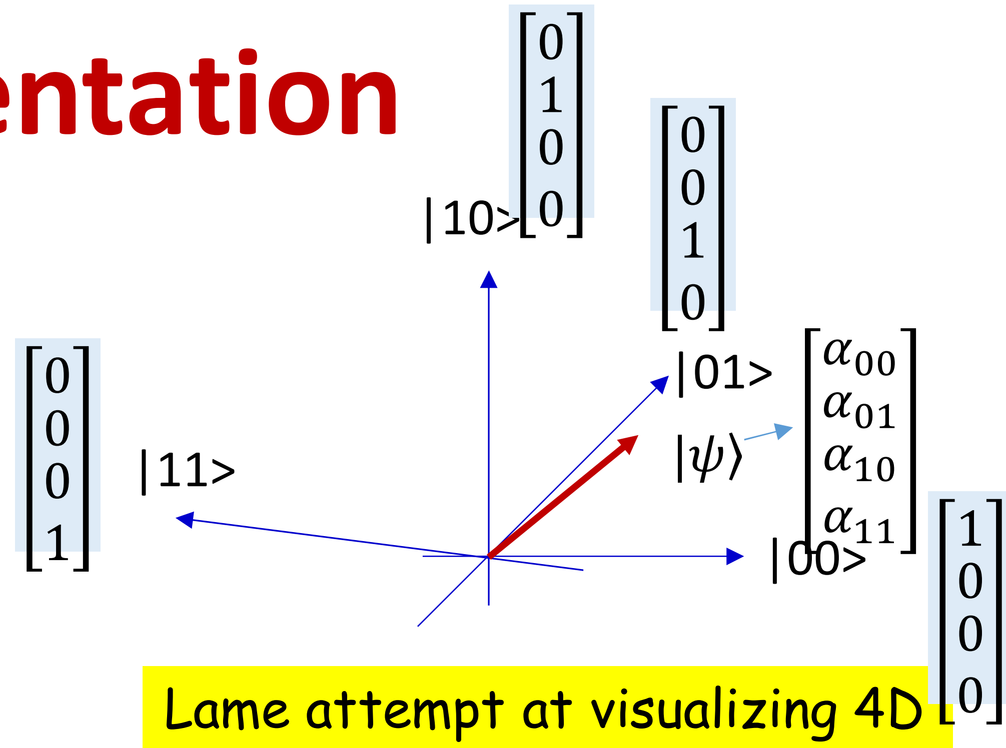


Lame attempt at visualizing 4D

- $|\psi\rangle = \alpha_{00\ldots0}|00\ldots0\rangle + \alpha_{00\ldots1}|00\ldots1\rangle + \cdots + \alpha_{11\ldots1}|11\ldots1\rangle$
- The $\alpha_{00\ldots0}$ terms are all *complex!*
  - *Thanks Heisenberg*
- Cannot even visualize a single qubit
  - One way to visualize 1 qubit: bloch sphere
  - Doesn't really scale

A vector representation





- Write the phasors as a regular vector
- One-qubit phasor

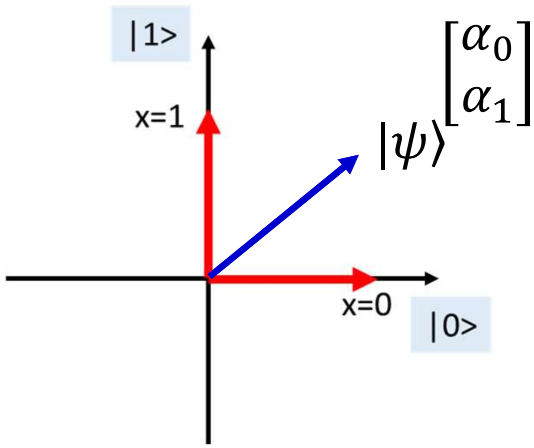
$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix}$$

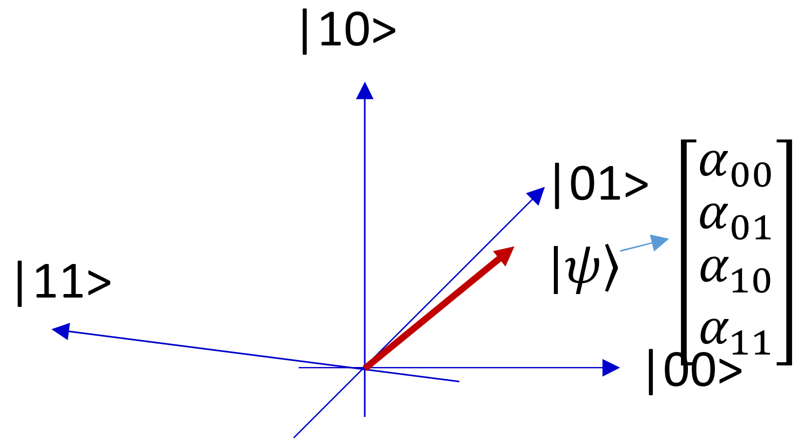
- Two qubit phasor

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle = \begin{bmatrix}\alpha_{00}\\ \alpha_{01}\\ \alpha_{10}\\ \alpha_{11}\end{bmatrix}$$

- The α terms are all *complex!*
  - *Thanks Heisenberg*
- Note the phasors for the basis bit patterns

The problem of measurement





Lame attempt at visualizing 4D

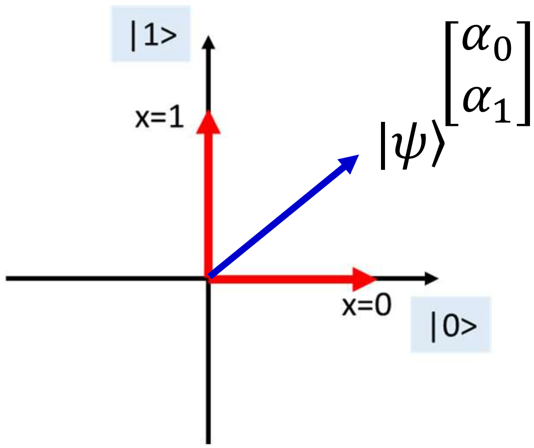
- You cannot measure the phasor
  - It will collapse to one of the bases
- One qubit:

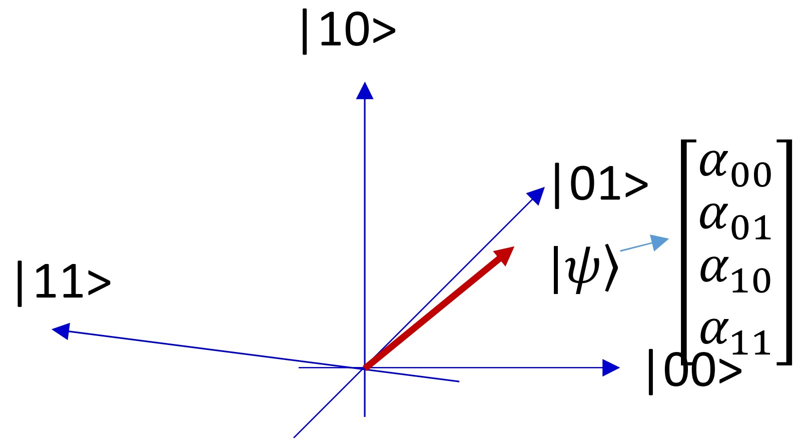
$$\alpha_0|0\rangle + \alpha_1|1\rangle \rightarrow \begin{cases} |0\rangle \ with\ probability\ |\alpha_0|^2 \\ |1\rangle \ with\ probability |\alpha_1|^2 \end{cases}$$

- Two qubits:

$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \rightarrow \begin{cases} |00\rangle \ with\ probability\ |\alpha_{00}|^2 \\ |01\rangle \ with\ probability\ |\alpha_{01}|^2 \\ |10\rangle \ with\ probability\ |\alpha_{10}|^2 \\ |11\rangle \ with\ probability\ |\alpha_{11}|^2 \end{cases}$$

The problem of measurement





Lame attempt at visualizing 4D

Clearly: $\sum|\alpha|^2 = 1$

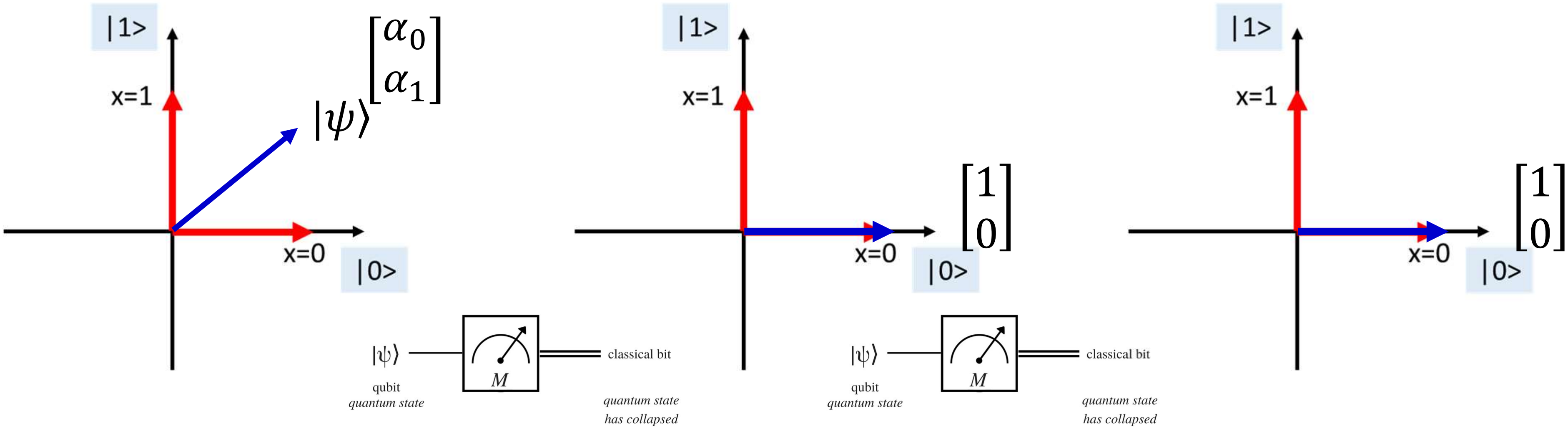
- You cannot measure the phasor
  - It will collapse to one of the bases
- One qubit:

$$\alpha_0|0\rangle + \alpha_1|1\rangle \rightarrow \begin{cases} |0\rangle\ with\ probability\ |\alpha_0|^2 \\ |1\rangle\ with\ probability |\alpha_1|^2 \end{cases}$$

- Two qubits:

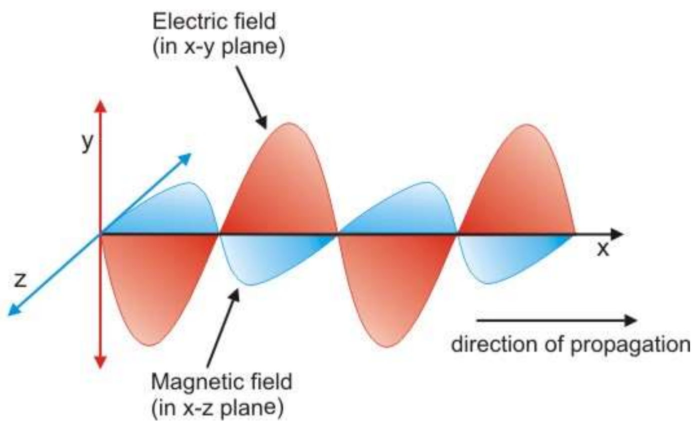
$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \rightarrow \begin{cases} |00\rangle\ with\ probability\ |\alpha_{00}|^2 \\ |01\rangle\ with\ probability\ |\alpha_{01}|^2 \\ |10\rangle\ with\ probability\ |\alpha_{10}|^2 \\ |11\rangle\ with\ probability\ |\alpha_{11}|^2 \end{cases}$$

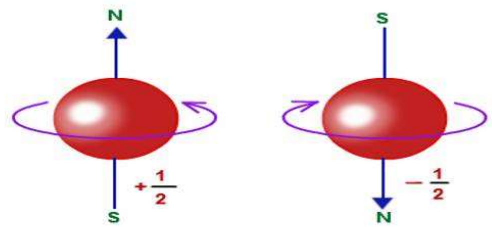
Measurement *fixes* the qubit

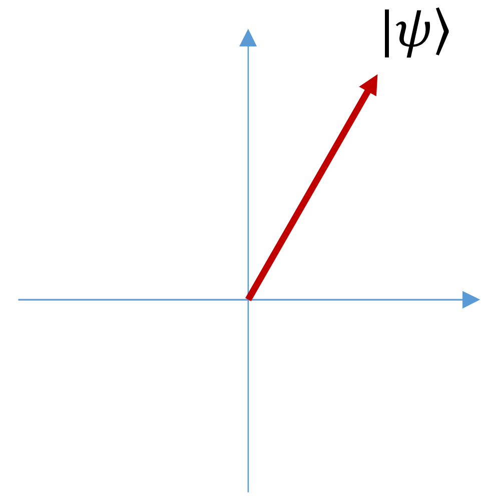


- First measurement: $|\psi\rangle$ → $|0\rangle$ with P = $|\alpha_0|^2$
- Second measurement: $|0\rangle$ → $|0\rangle$ with P = 1
- Third measurement: $|0\rangle$ → $|0\rangle$ with P = 1
- …

The phasors are unique, but the representation is not





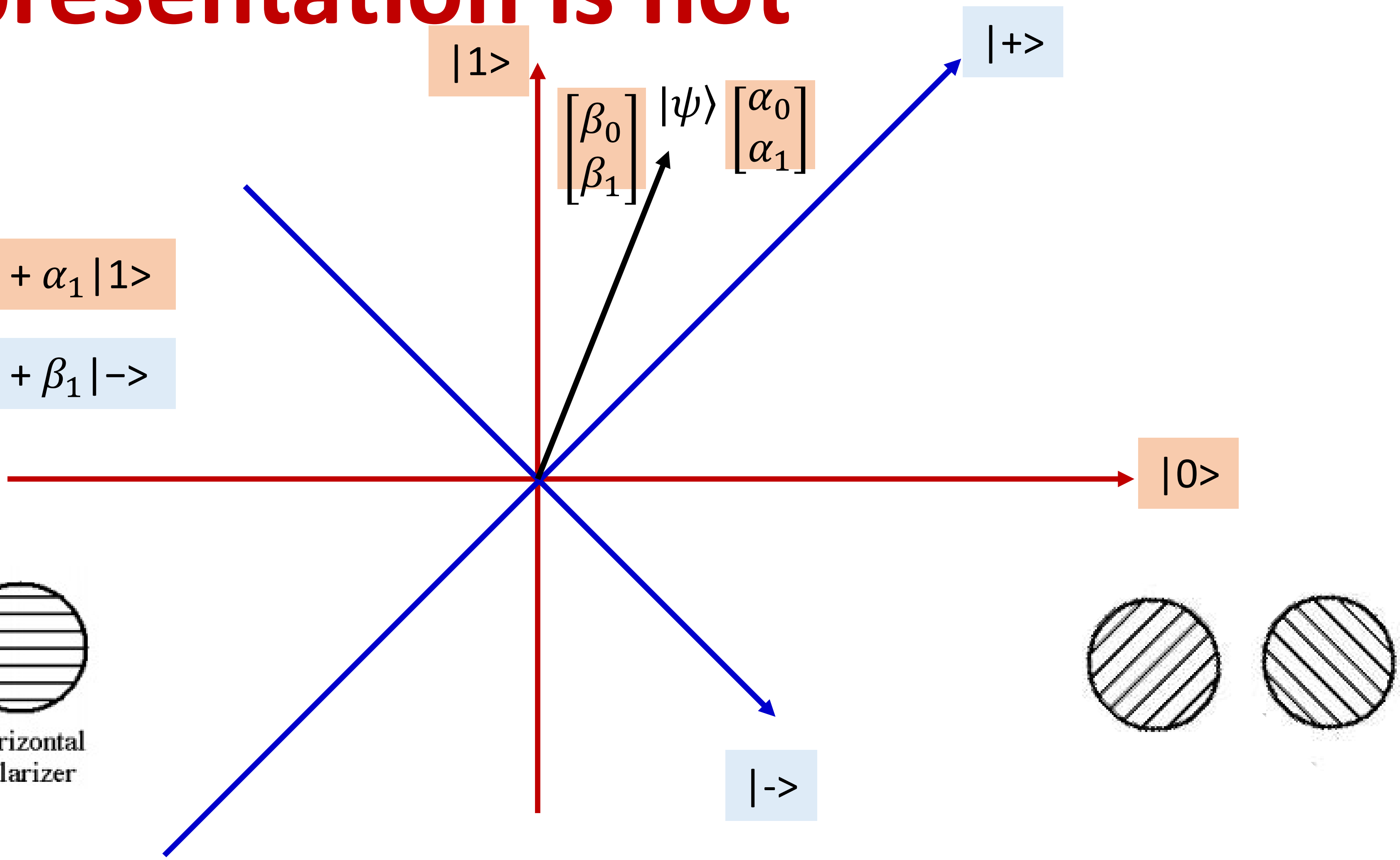


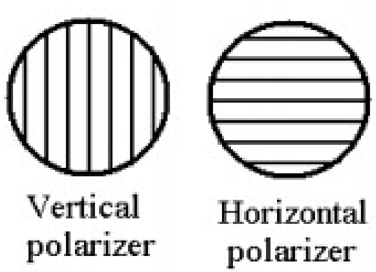
- No absolute definition of direction or sign
    - But the state of the system is well defined!
    - The space is defined, and the direction of the (physical) phasor is well defined
- The actual representation depends on the bases used
    - Only restriction: the bases must be orthogonal

The phasors are unique, the representation is not

$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$

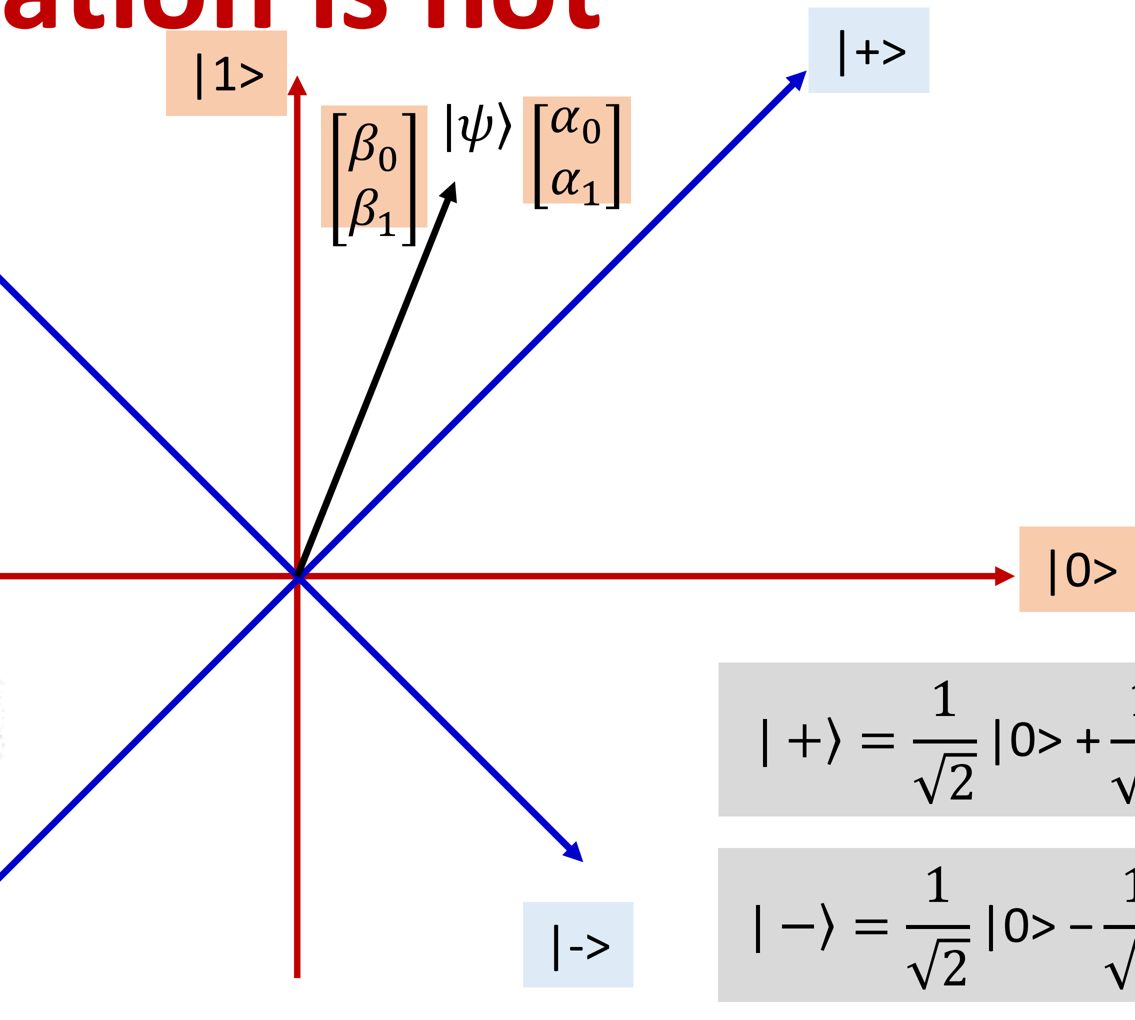
$|\psi\rangle = \beta_0 |+\rangle + \beta_1 |-\rangle$





- The representation depends on the bases
  - Think orientation of your polarized glasses..

The phasors are unique, the representation is not



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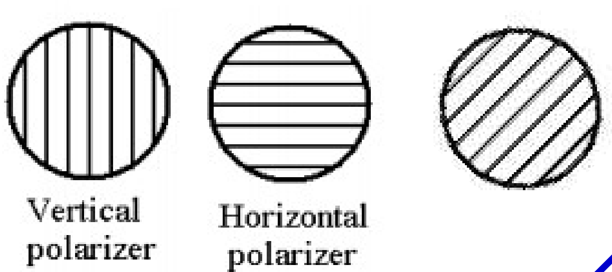


$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

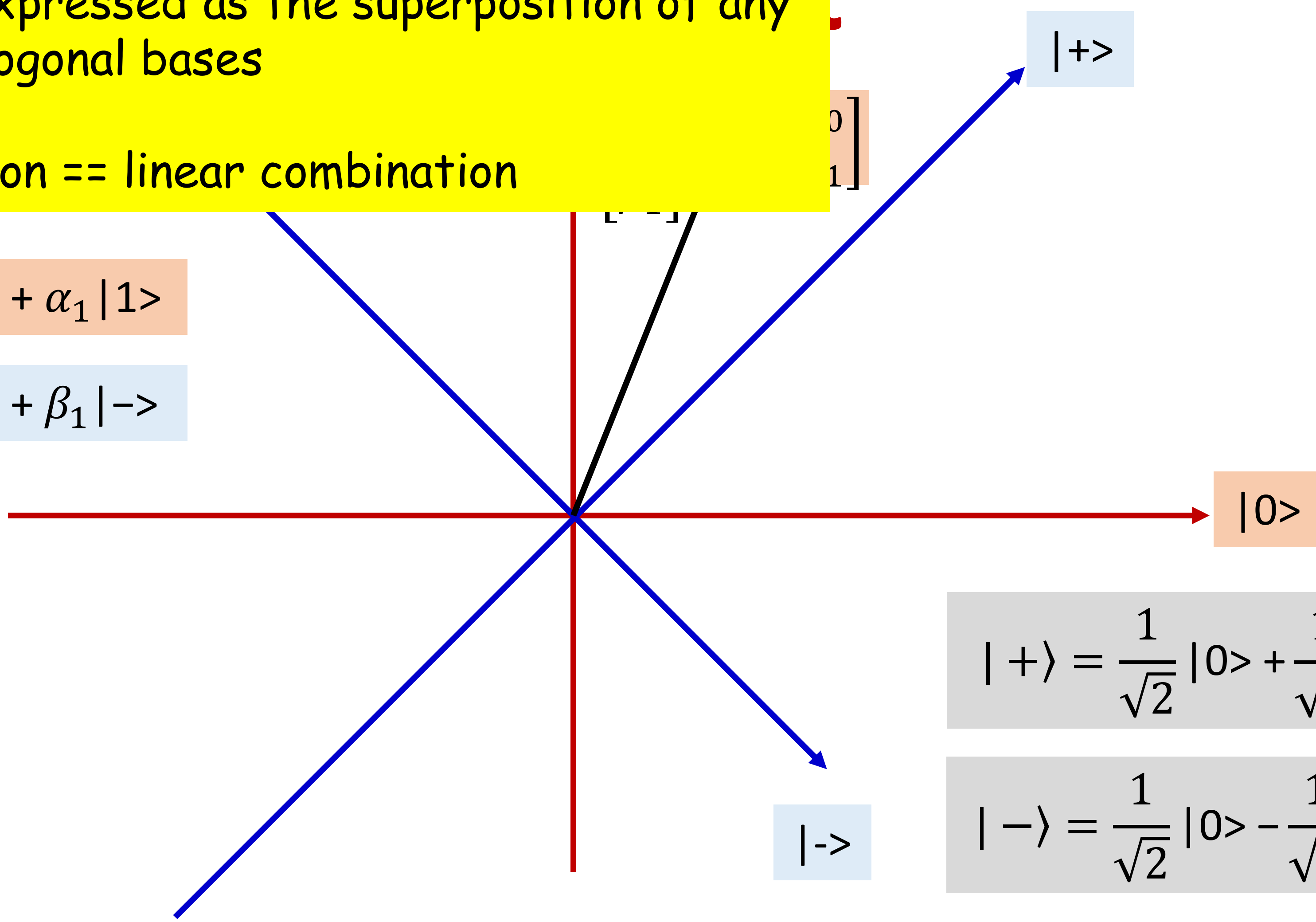
- The representation depends on the bases
  - Think orientation of your polarized glasses..

ue, the

The space of phasors is a complex Hilbert space
A qubit is a vector on a unit sphere in this space

It can be expressed as the superposition of any set of orthogonal bases

Superposition == linear combination



$|\psi\rangle = \alpha_0 |0> + \alpha_1 |1>$

$|\psi\rangle = \beta_0 |+> + \beta_1 |->$

$| +\rangle = \frac{1}{\sqrt{2}} |0> + \frac{1}{\sqrt{2}} |1>$

$| -\rangle = \frac{1}{\sqrt{2}} |0> - \frac{1}{\sqrt{2}} |1>$

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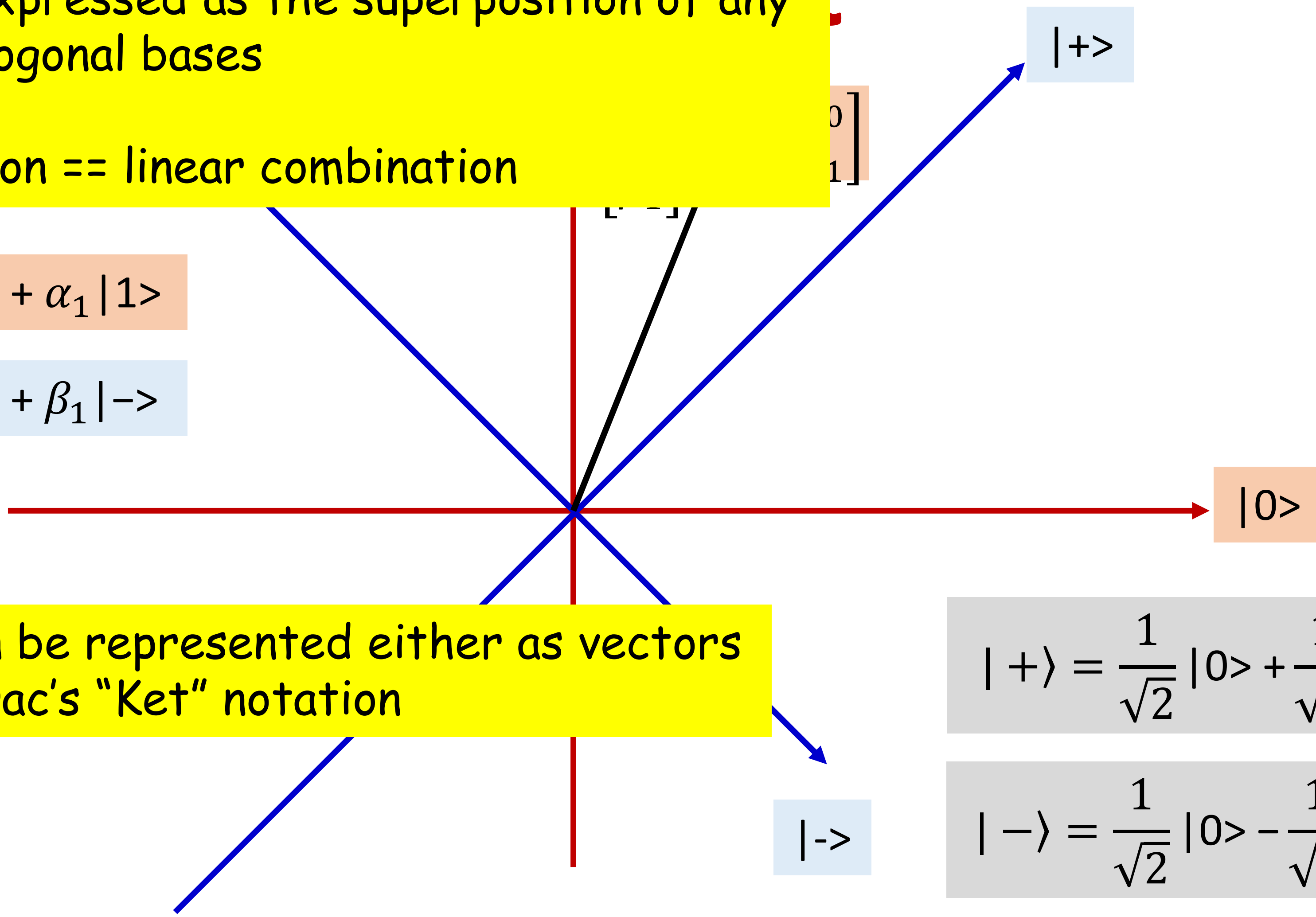


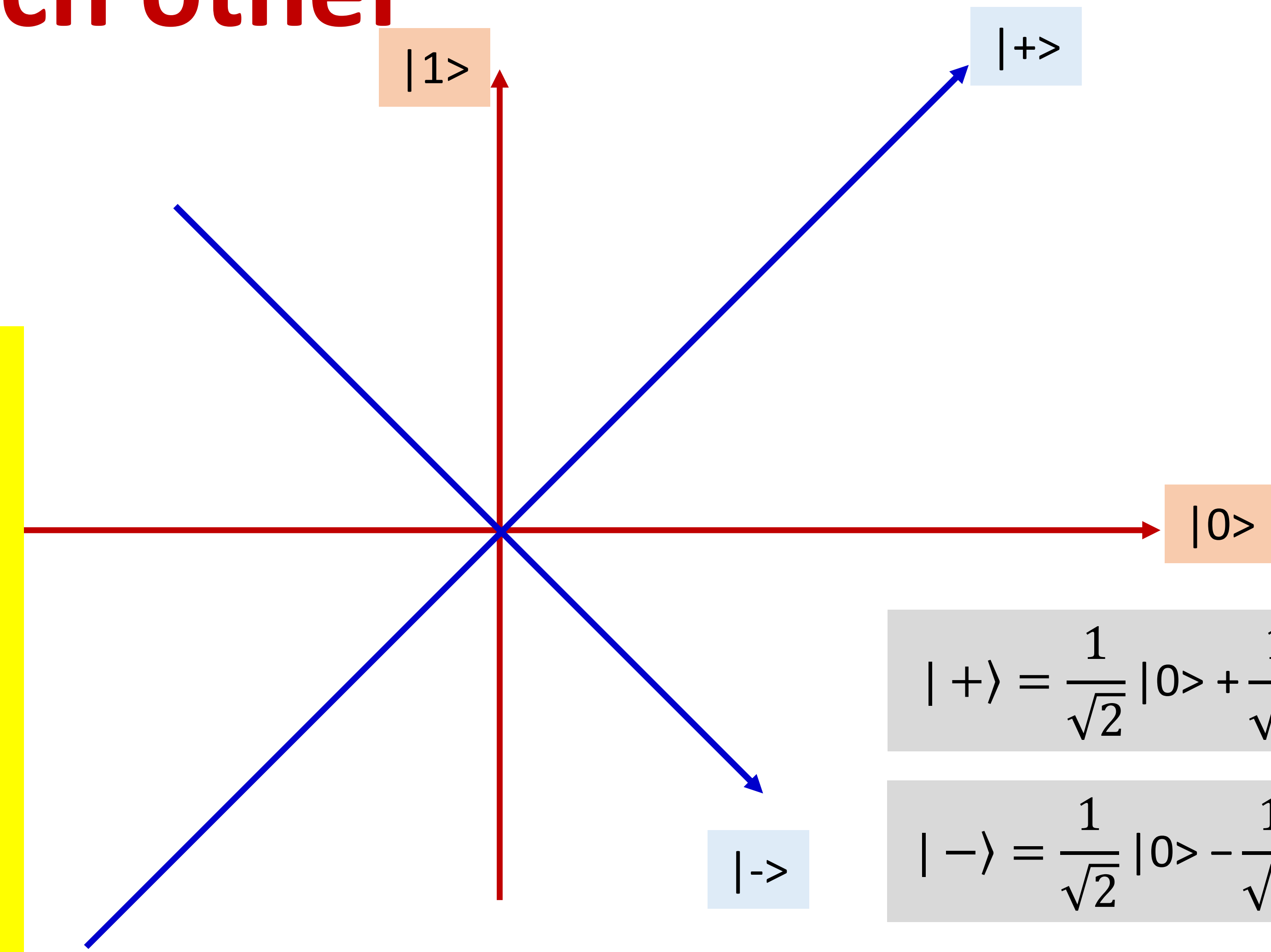
Phasors can be represented either as vectors or using Dirac's "Ket" notation

$$|+\rangle = \frac{1}{\sqrt{2}} |0> + \frac{1}{\sqrt{2}} |1>$$

$$|-\rangle = \frac{1}{\sqrt{2}} |0> - \frac{1}{\sqrt{2}} |1>$$

- The representation depends on the bases
  - Think orientation of your polarized glasses..

Bases can be expressed in terms of each other



In bit bases:

$$|+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|-\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0> + \frac{1}{\sqrt{2}}|1>$$

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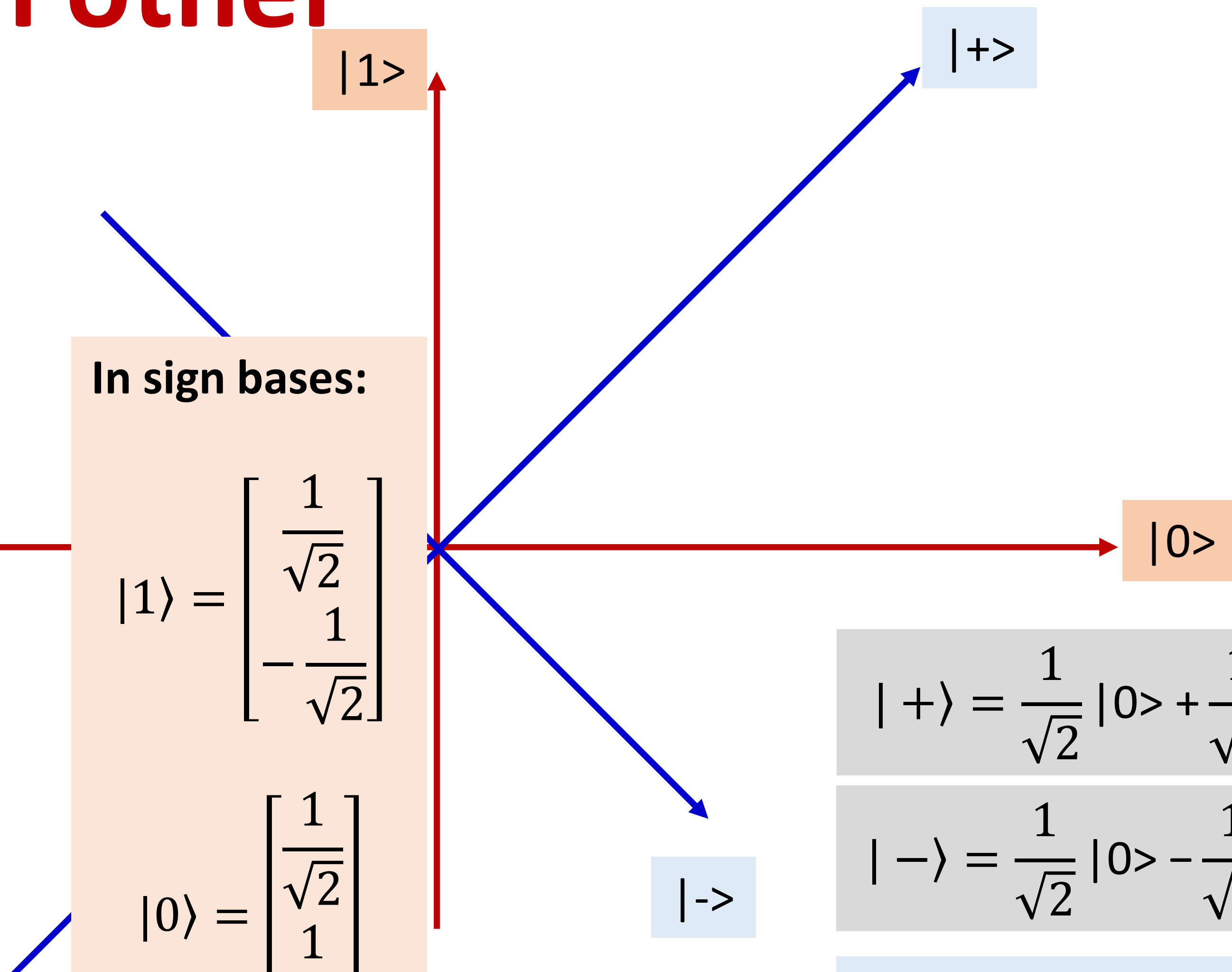

In bit bases:

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$$|-\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

In sign bases:

$$|1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

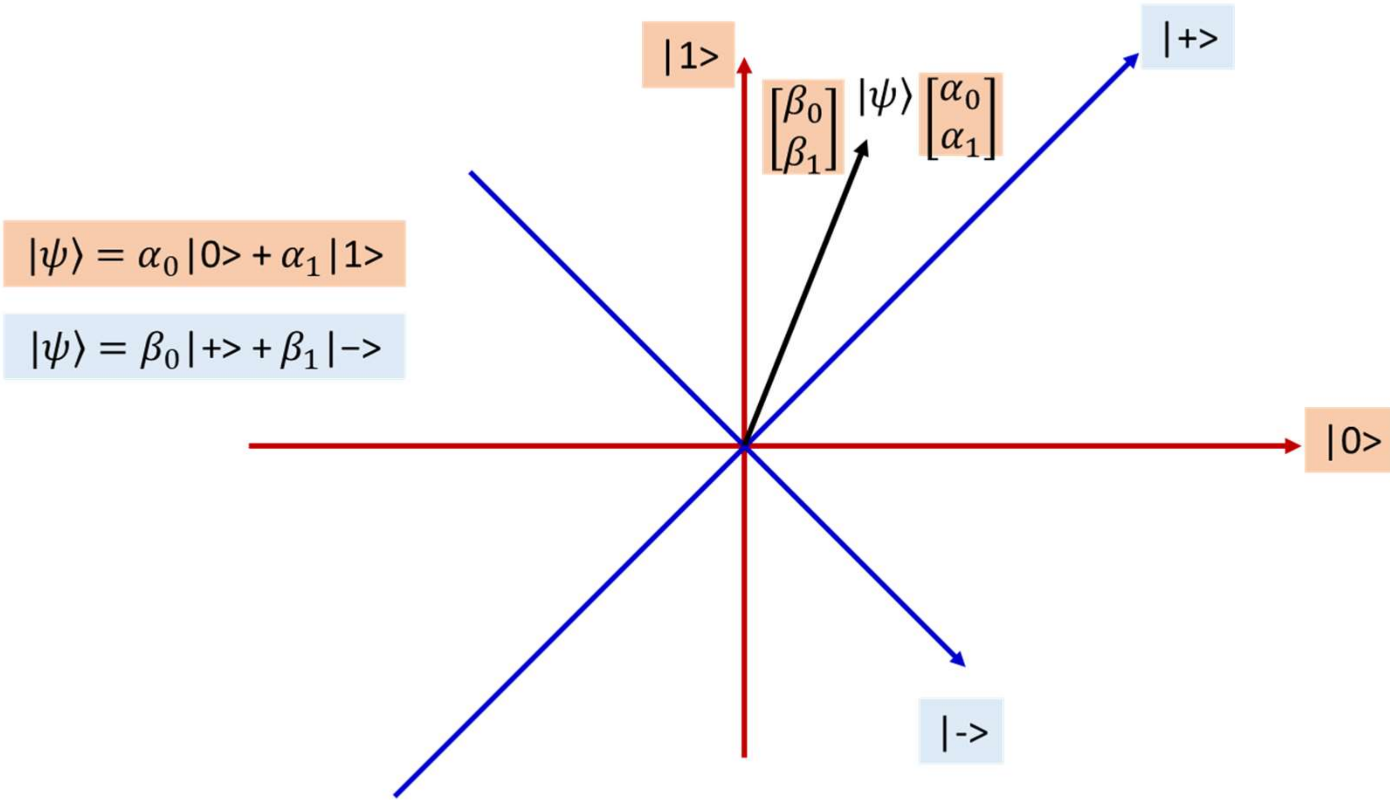
$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

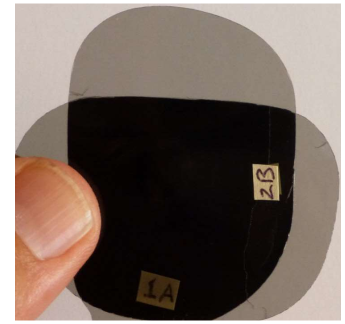
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$$

$$|1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

You can *measure* using either bases!!



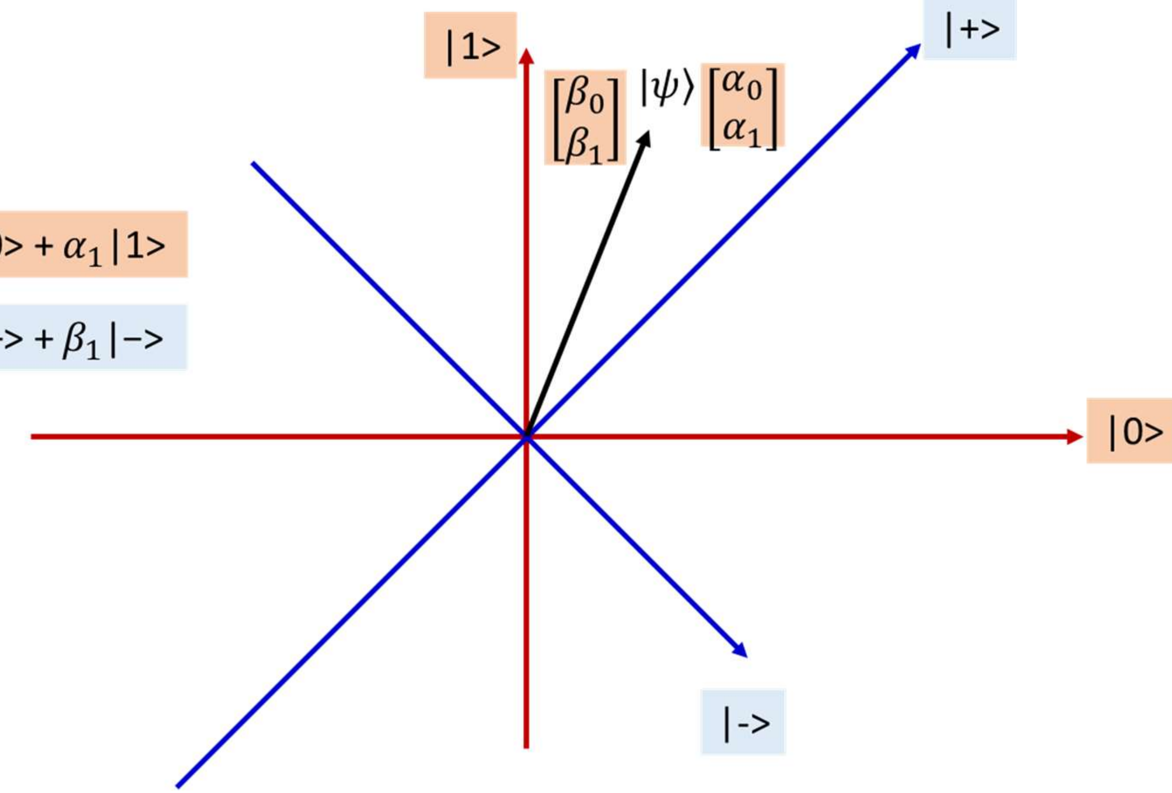


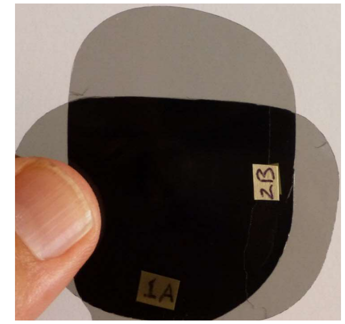
- What are P(|0>) andP(|1>)?
- What are P(|+>) andP(|−>)?

You can *measure* using either bases!!

$|\psi\rangle = \alpha_0|0> + \alpha_1|1>$

$|\psi\rangle = \beta_0|+> + \beta_1|->$





- What are P(|0>) andP(|1>) ?
- What are P(|+>) andP(|−>) using α_0 and α_1?

So what is measurement

- Measurement *projects* the phasor on the basis with a probability that is the square of the length of the projection
  - Using bit basis representation, but measuring on sign basis:

$$P(|+>)= \left| \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}^H \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right|^2$$

$$P(|->)= \left| \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}^H \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right|^2$$

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What will be P(|+>) and P(|->) using the sign bases for representation?

$$P(|->)=\left|\begin{bmatrix}\alpha_0\\\alpha_1\end{bmatrix}^H\begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}\right|^2$$

What will be P(|0>) and P(|1>) using the sign bases for representation?

So what is measurement

- Measurement *projects* the phasor on the basis with a probability that is the square of the length of the projection
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$$P(|+>)=\left|\begin{bmatrix}\alpha_0\\\alpha_1\end{bmatrix}^H\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix}\right|^2$$

P(basis) is simply the square of the cosine of the angle between the phasor and the basis

$$P(|->)=\left|\begin{bmatrix}\alpha_0\\\alpha_1\end{bmatrix}^H\begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}\right|^2$$

What will be P(|+>) and P(|->) using the sign bases for representation?

What will be P(|0>) and P(|1>) using the sign bases for representation?

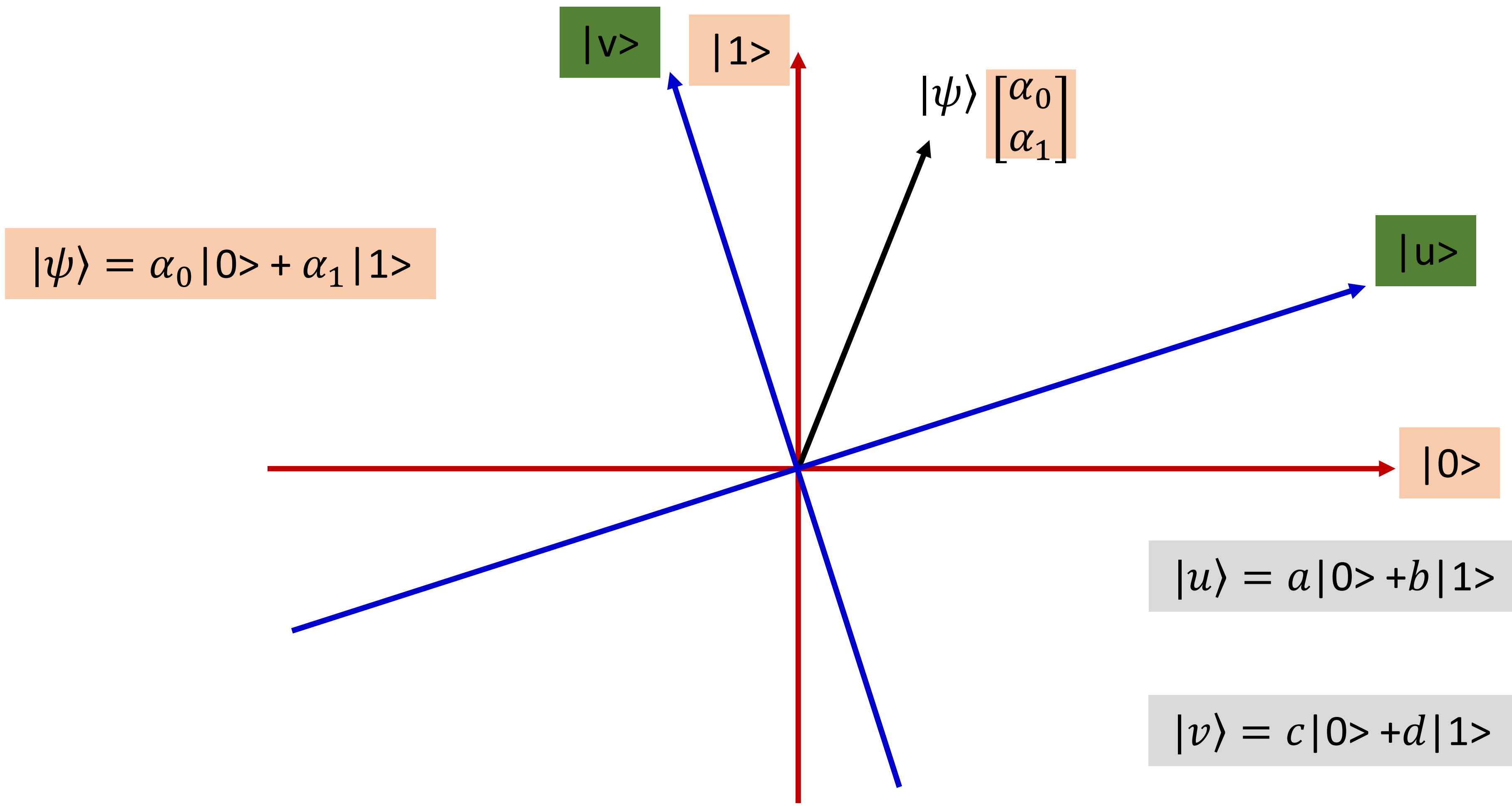
Some basic math

- The projection of a complex vector a on a complex vector b is given by

$$a^H b = \sum_i a_i^* b$$

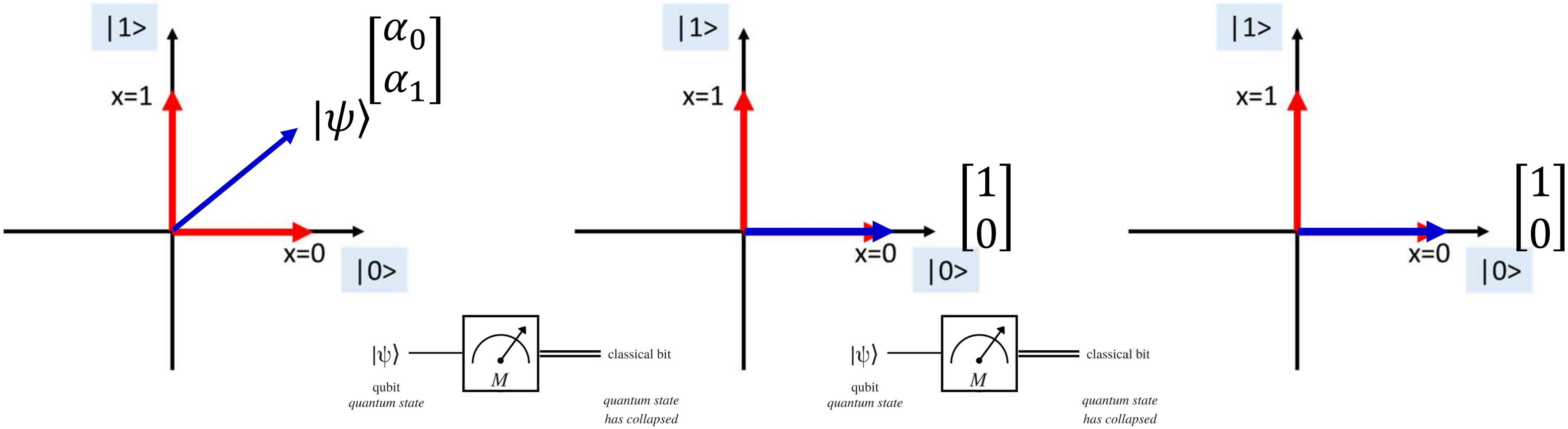
- For unit vectors it is the cosine of the angle between them
- For any basis, the probability of measuring that basis is the square of the cosine between the phasor and the basis

A different basis…



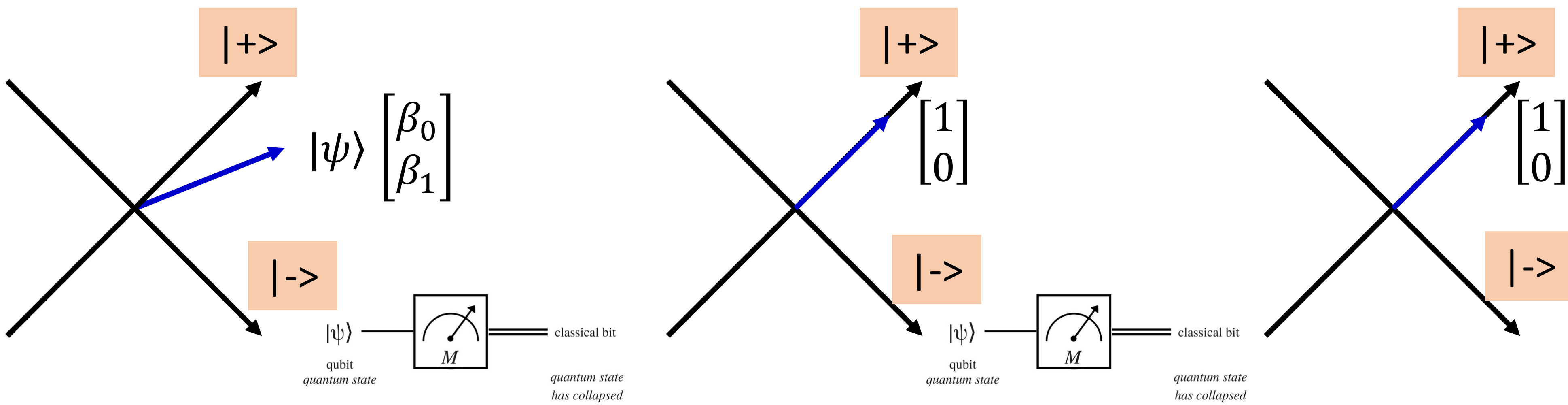
- What would P(|u>) and P(|v>) be?

Measurement *simplifies* life



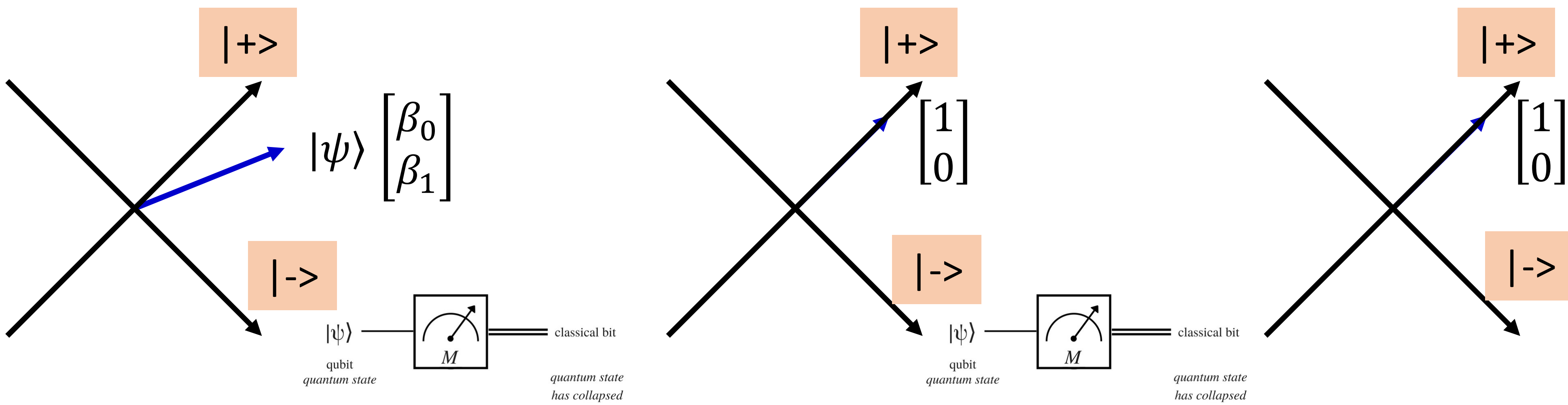
- By fixing the value

Measurement *simplifies* life



- By fixing the value

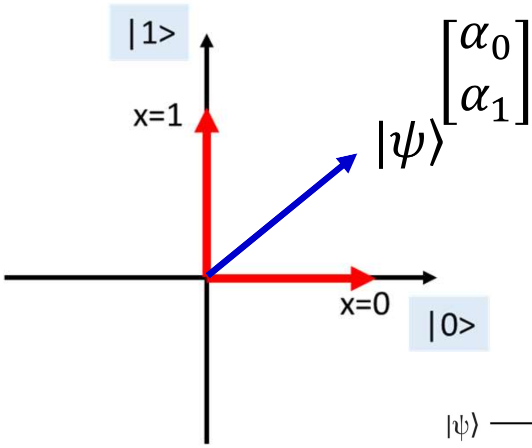
Measurement *simplifies* life

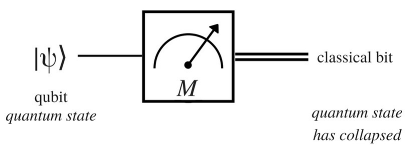


- By fixing the value

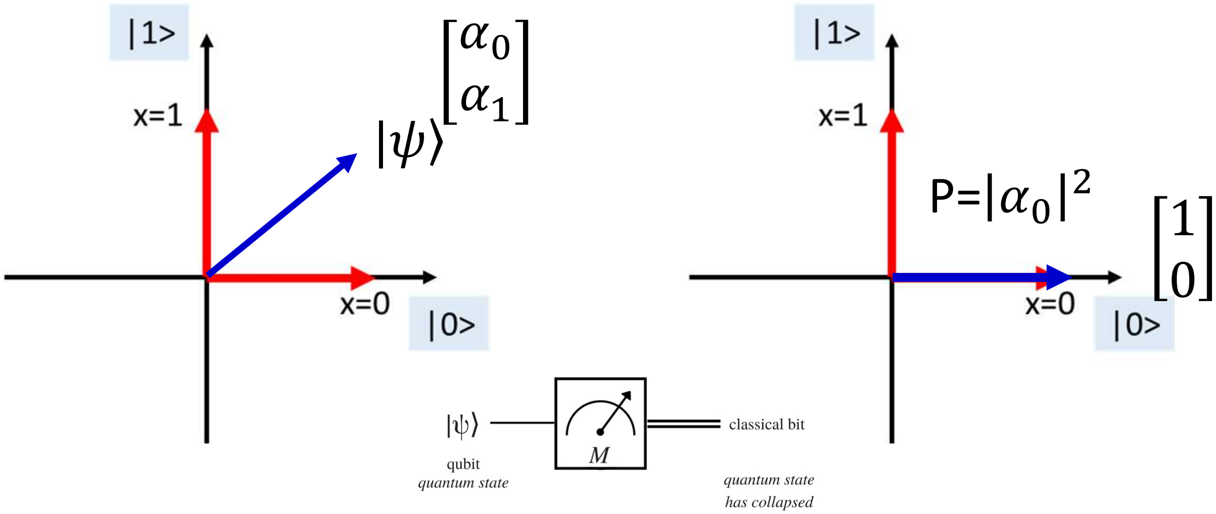
- Or does it?

Measurement *simplifies* life

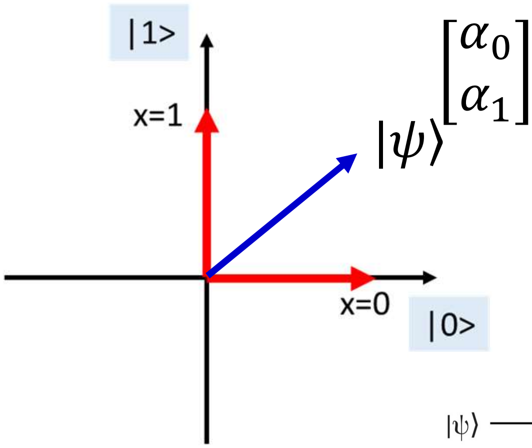


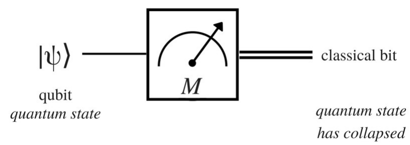


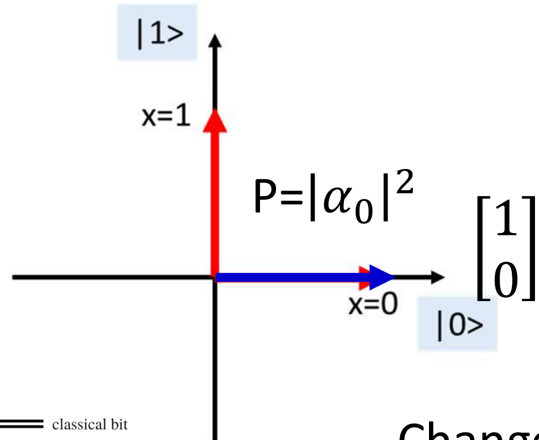
Measurement *simplifies* life



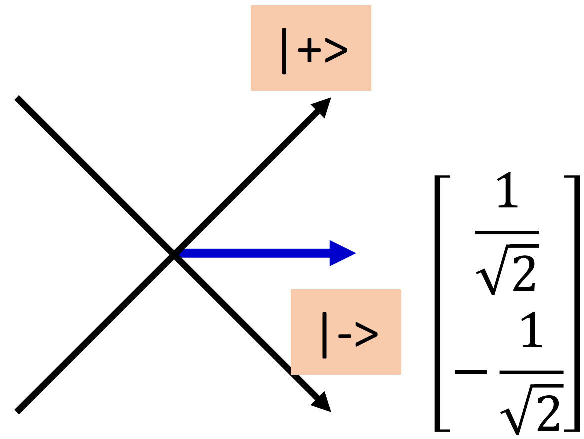
Measurement *simplifies* life



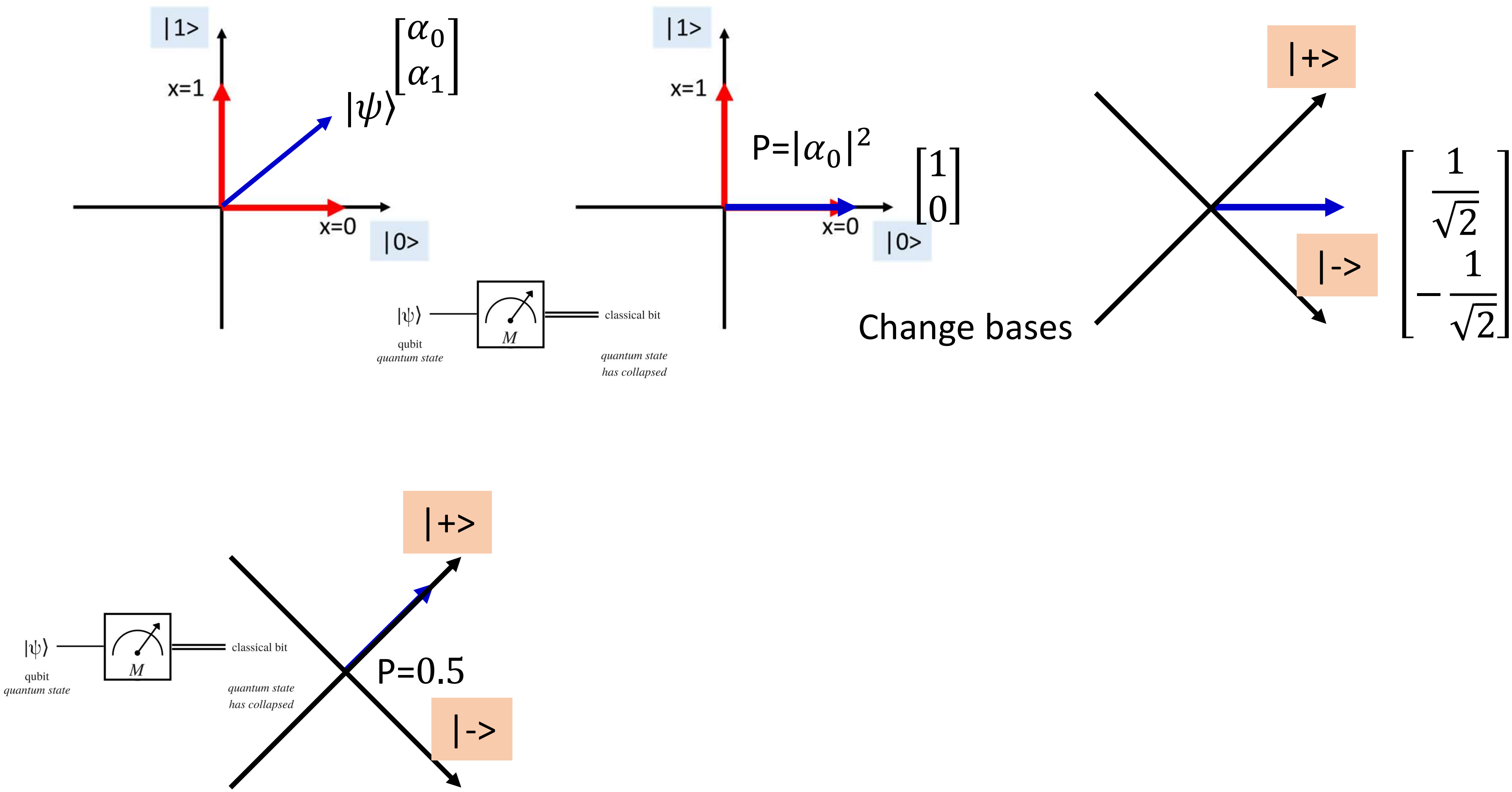




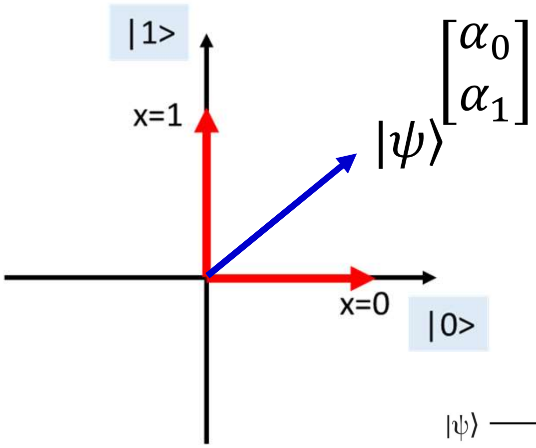
Change bases

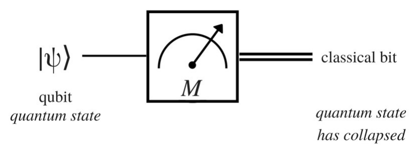


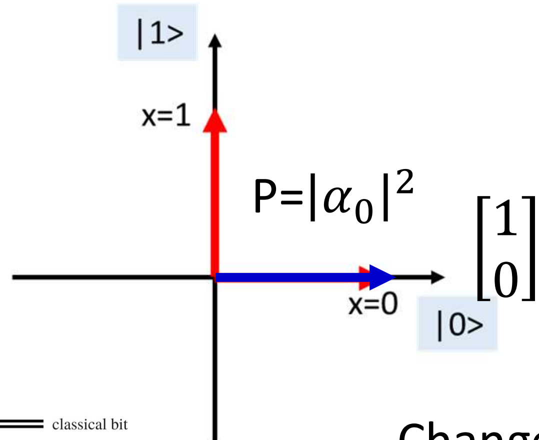
Measurement *simplifies* life



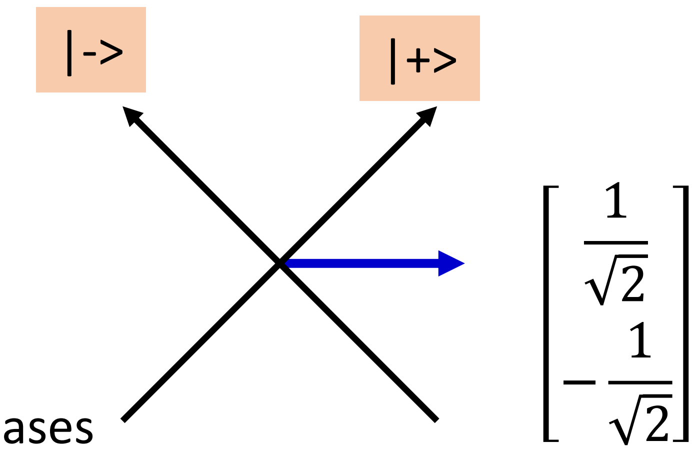
Measurement *simplifies* life

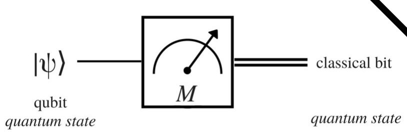


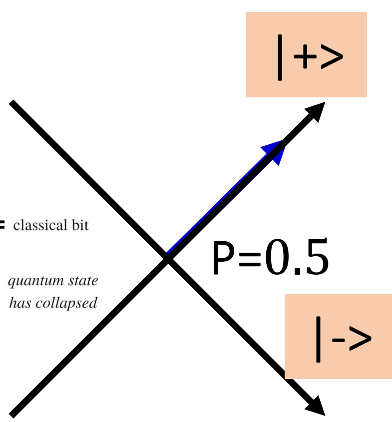




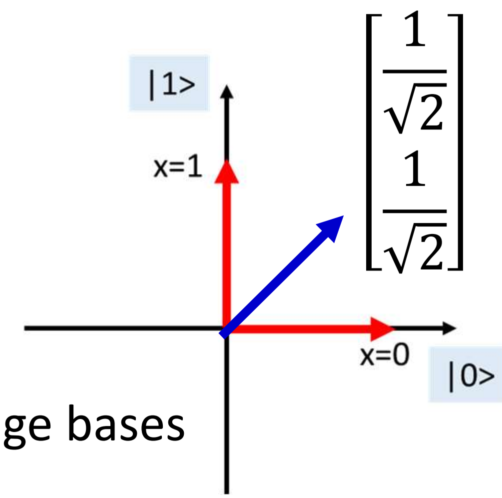
Change bases



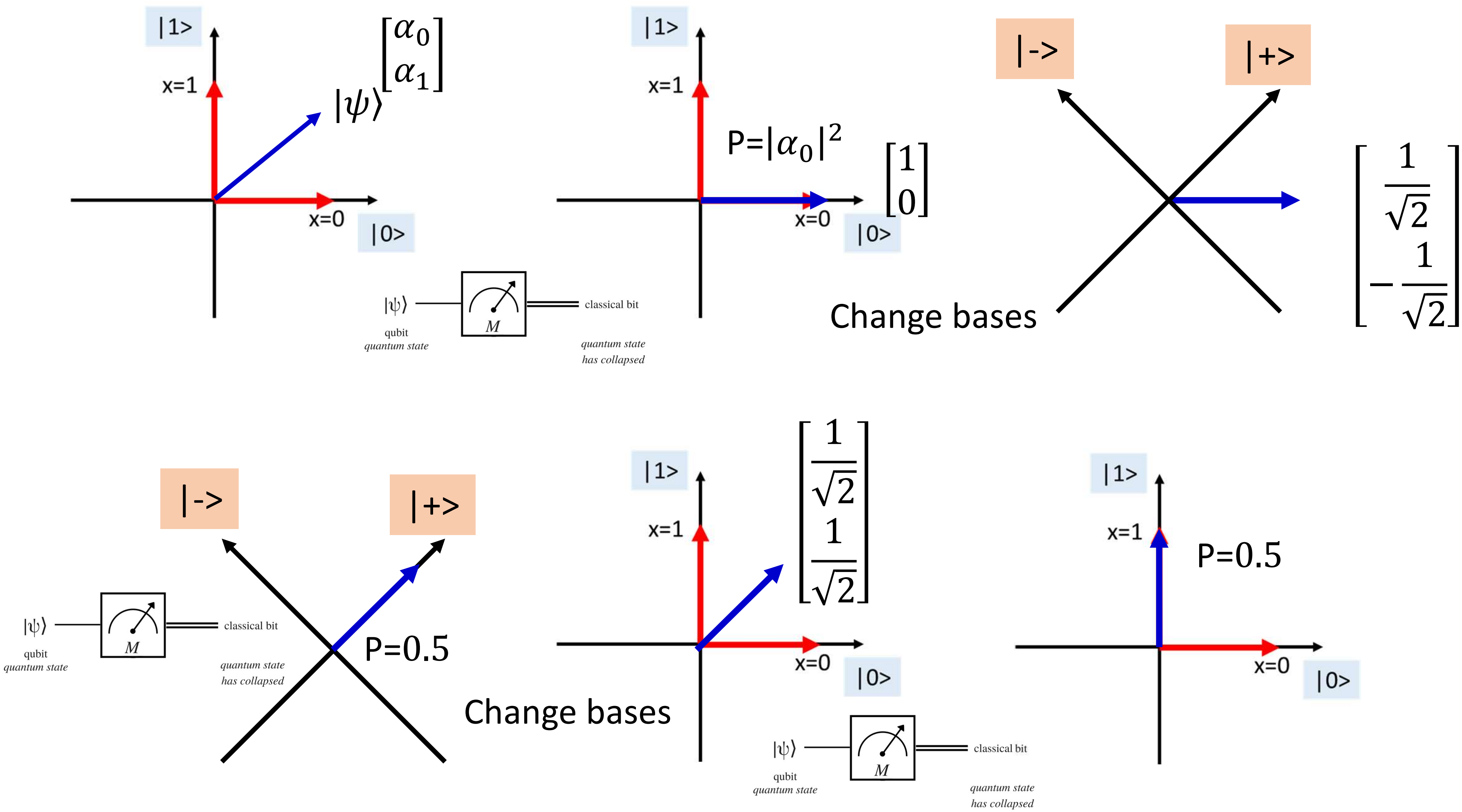




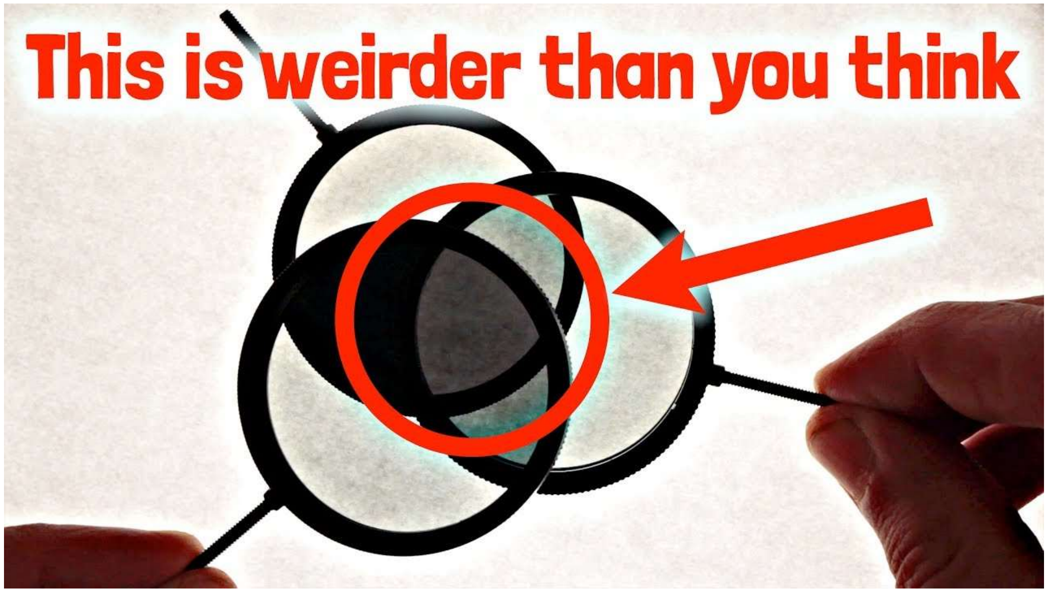
ange bases



Measurement *simplifies* life



The world isn't what you think it is!!!



- Repeated measurements with different bases can completely alter reality!!!